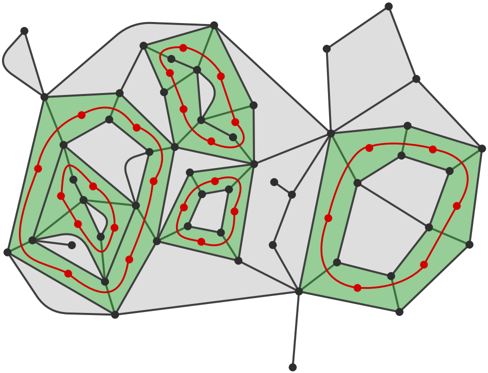
Séminaire Philippe Flajolet, IHP, Paris, 03-12-2015

The peeling process on random planar maps with loops

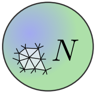
Timothy Budd



Based on arXiv:1506.01590 and arXiv:1512.xxxxx.

Niels Bohr Institute, University of Copenhagen
budd@nbi.dk, http://www.nbi.dk/~budd/

Motivation [Le Gall, Miermont, Borot, Bouttier, Guitter, Sheffield, Miller, . . .]

$\Big($  $, N^{-\frac{1}{4}} d_{\mathrm{gr}} \Big) \xrightarrow[N \to \infty]{\text{Gromov-Hausdorff}}$ 

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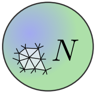

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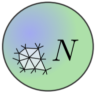

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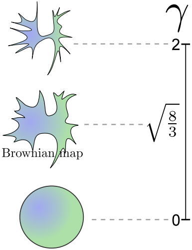
Liouville Quantum Gravity



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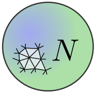

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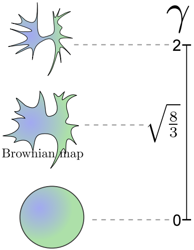
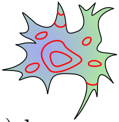
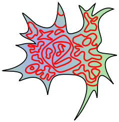




$O(n)$ loop model



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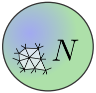

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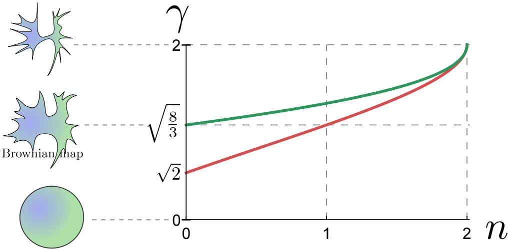

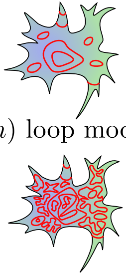

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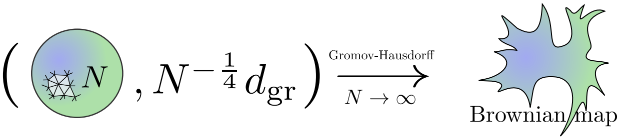

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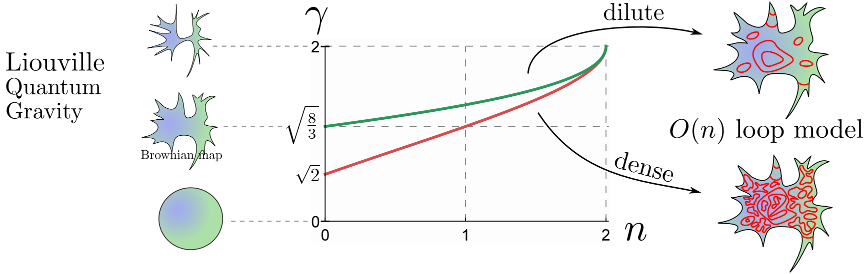

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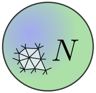

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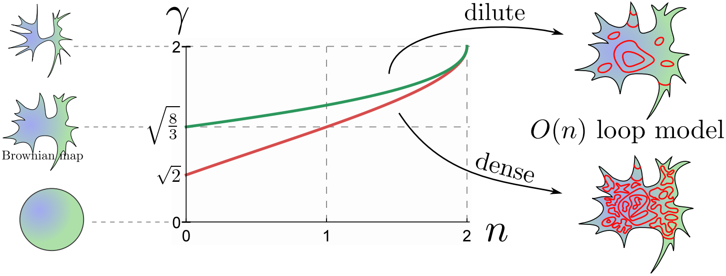

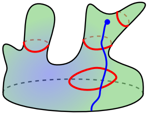

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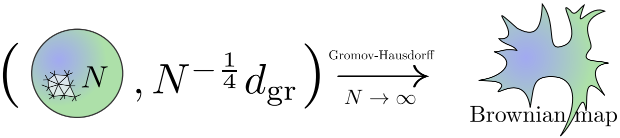

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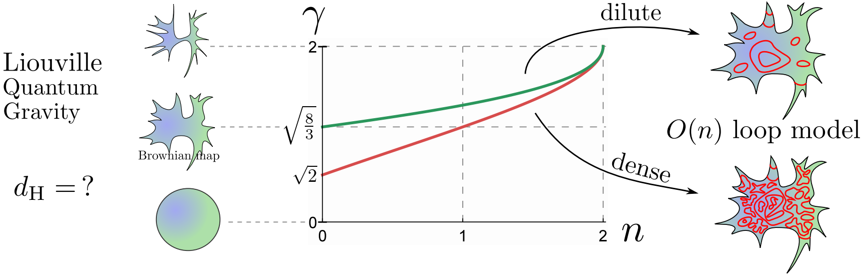

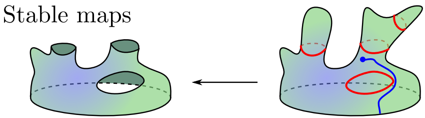

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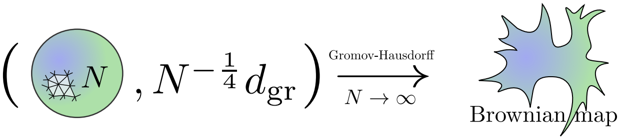

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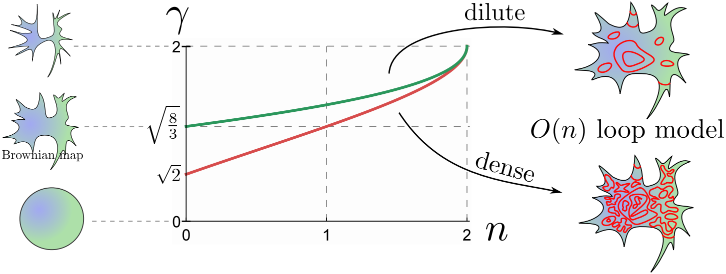
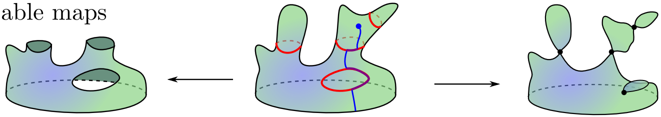
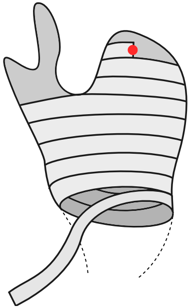


Stable maps



Peeling processes

- The peeling process lead to the first (heuristic) determination of the 2-point function of random triangulations. [Watabiki, Ambjørn,'95]



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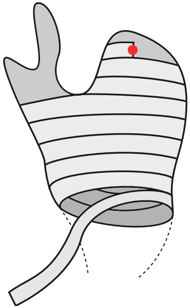

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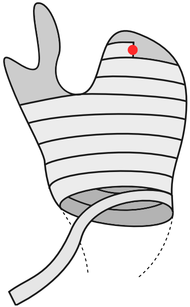

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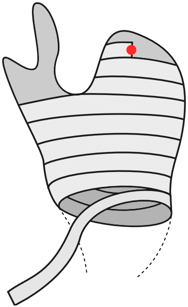

- This talk: in the case of Boltzmann planar maps, with general but controlled face degree, the peeling process gives a useful relation

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Boltzmann planar maps $\longleftrightarrow$ Random walks

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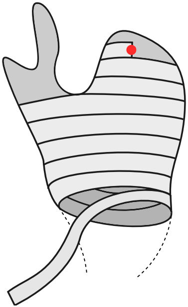

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$$\text{Boltzmann planar maps} \longleftrightarrow \text{Random walks} \xrightarrow{\text{scaling limit}} \text{Stable processes}$$

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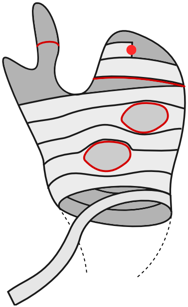

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Boltzmann planar maps $\longleftrightarrow$ Random walks $\xrightarrow{\text{scaling limit}}$ Stable processes

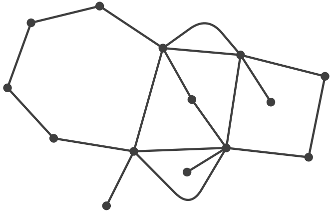
Loop-decorated planar maps $\longleftrightarrow$ Partially reflected random walks $\xrightarrow{\text{scaling limit}}$ Partially reflected stable processes

Outline

- Preliminaries
  - Boltzmann planar maps
  - The $O(n)$ model: Boltzmann loop-decorated maps
  - Gasket decomposition
- Peeling process
  - Boltzmann planar maps $\longleftrightarrow$ Random walks
  - Boltzmann loop-decorated planar maps $\longleftrightarrow$ Partially reflected random walks
- Scaling limit
  - Convergence of perimeter to a self-similar Markov process
  - Law of integral
  - Potential application: distance with shortcuts on loops

Boltzmann planar maps

- Let $\mathfrak{m} \in \mathcal{M}^{(l)}$ be a bipartite rooted planar map with root face degree $2l$.



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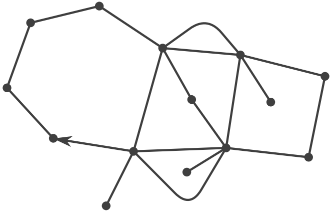

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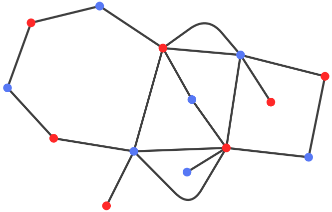

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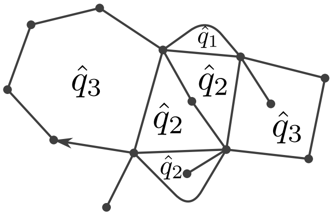

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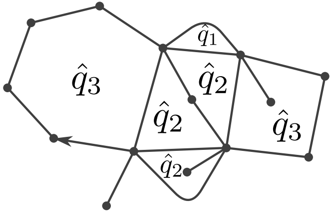

Boltzmann planar maps

- Let $\mathfrak{m} \in \mathcal{M}_{\bullet}^{(l)}$ be a bipartite rooted planar map with root face degree $2l$ and a marked vertex.
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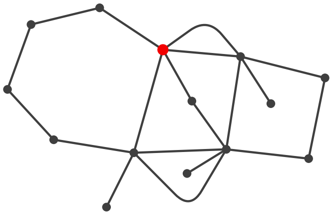
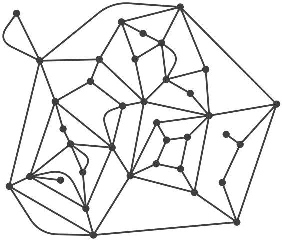


Boltzmann loop-decorated maps

- A rigid loop-decorated map $(\mathfrak{m}, L) \in \mathcal{LM}^{(l)}$ is a rooted planar map with root face degree $2l$ and a set L of loops on the dual map.



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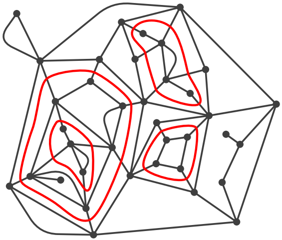

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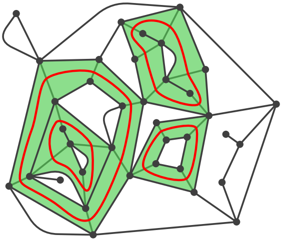

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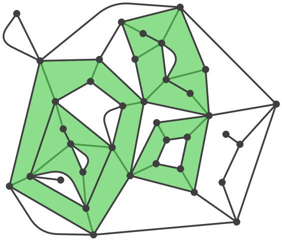

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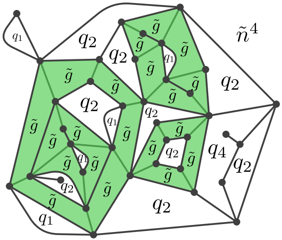

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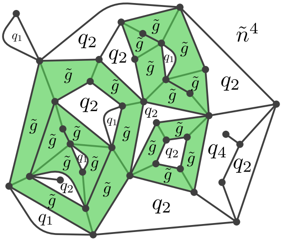
  Gives rise to the $(\mathbf{q}, \tilde{g}, \tilde{n})$-*Boltzmann loop-decorated map*.



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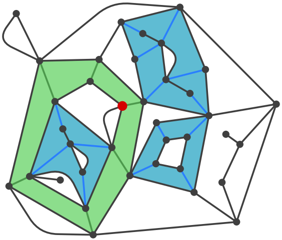
  Gives rise to the $(\mathbf{q}, \tilde{g}, \tilde{n})$-*Boltzmann loop-decorated map*.
- In the presence of a marked vertex it is convenient to distinguish *separating* from *non-separating* loops. [Borot, Bouttier,'15]



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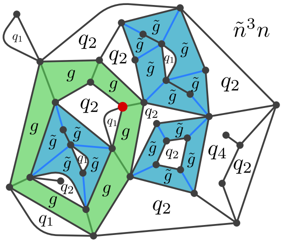
- $(\mathbf{q}, g, n, \tilde{g}, \tilde{n})$ *admissible* iff $F_\bullet^{(l)} := \sum_{(\mathfrak{m},L)\in\mathcal{LM}_\bullet^{(l)}} w_{\mathbf{q},g,n,\tilde{g},\tilde{n}}(\mathfrak{m}, L) < \infty$.

  Gives rise to the $(\mathbf{q}, g, n, \tilde{g}, \tilde{n})$-*Boltzmann loop-decorated map*.
- In the presence of a marked vertex it is convenient to distinguish *separating* from *non-separating* loops. [Borot, Bouttier,'15]



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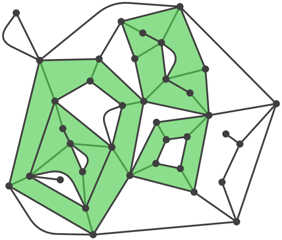

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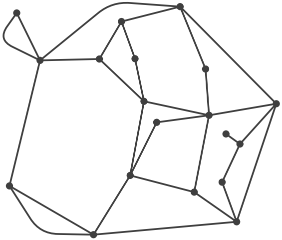

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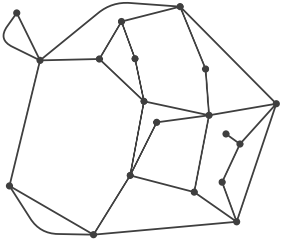

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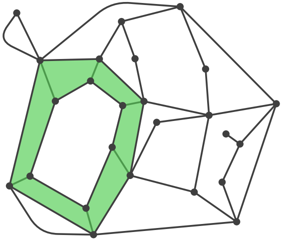

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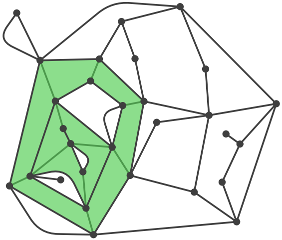

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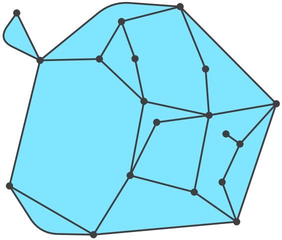

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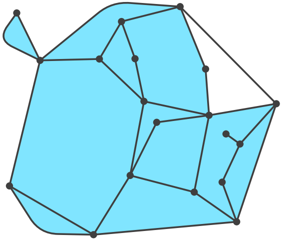

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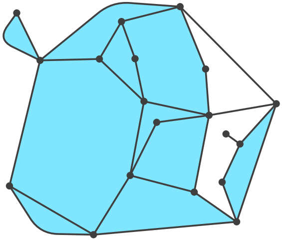

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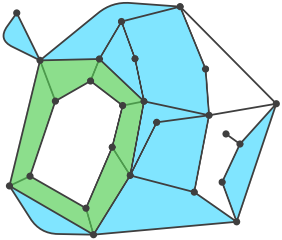

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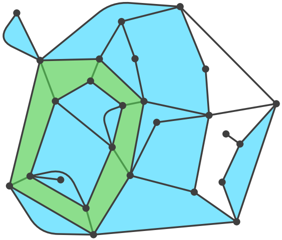

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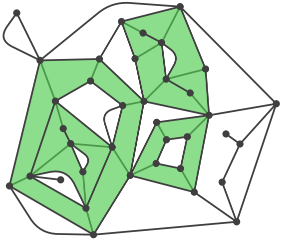

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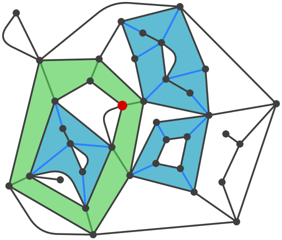

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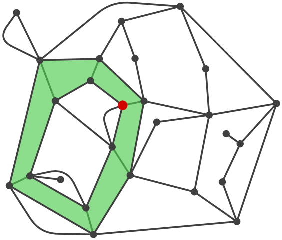

Peeling process on loop-decorated maps

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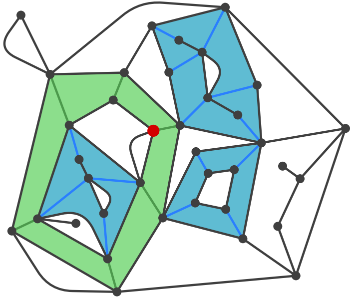

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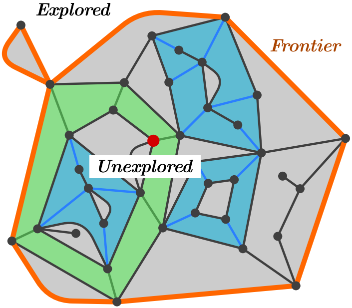

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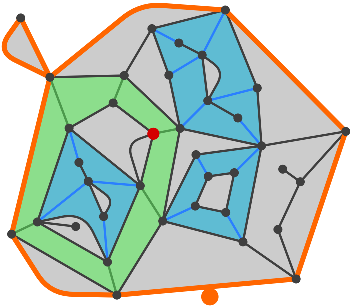

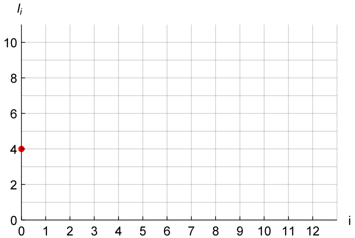

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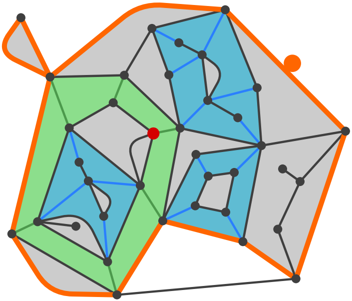

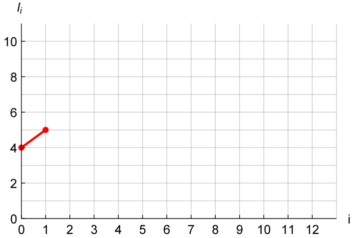

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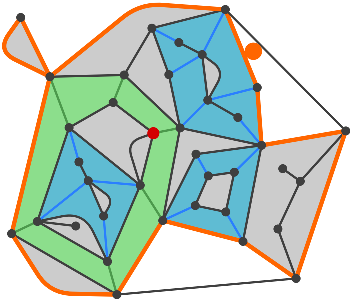

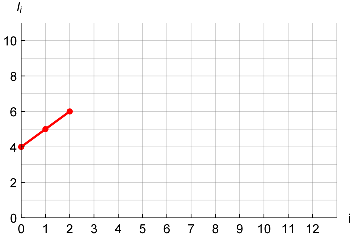

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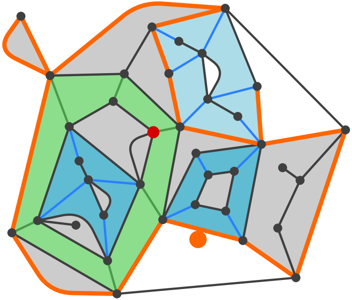

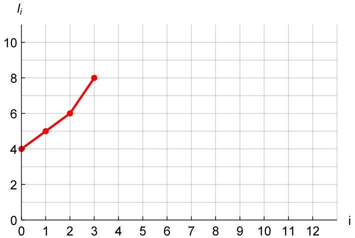

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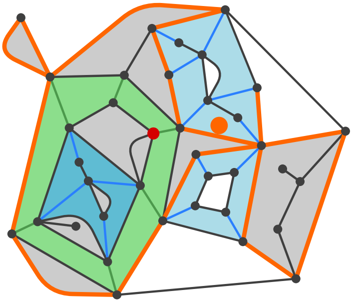

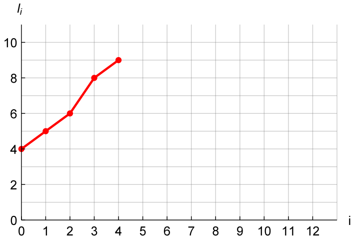

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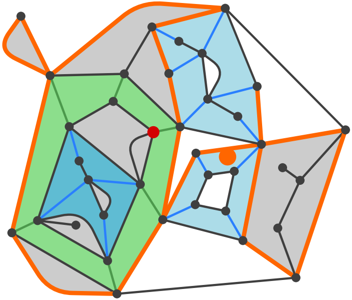

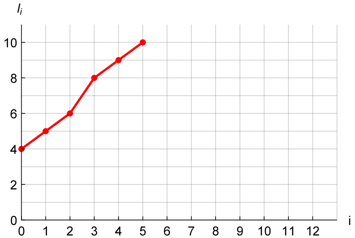

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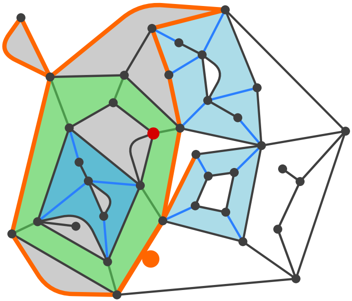

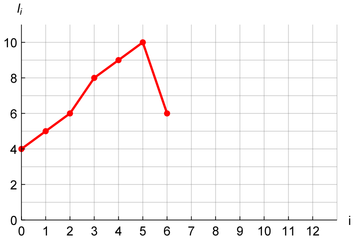

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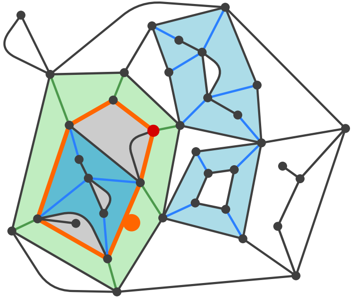

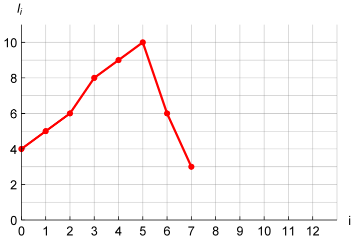

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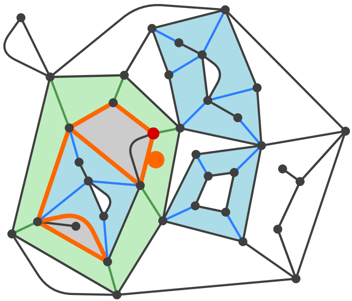

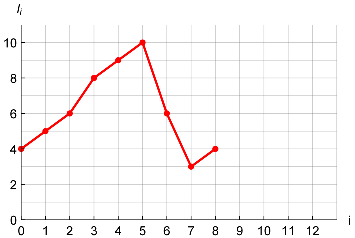

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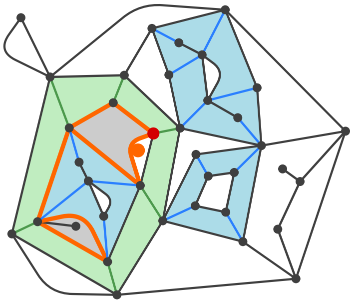

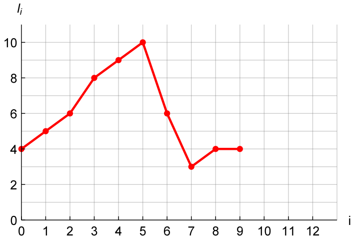

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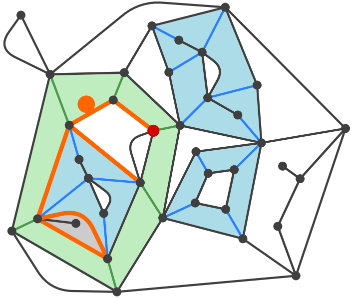

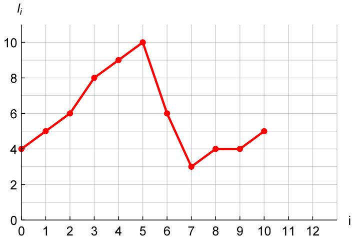

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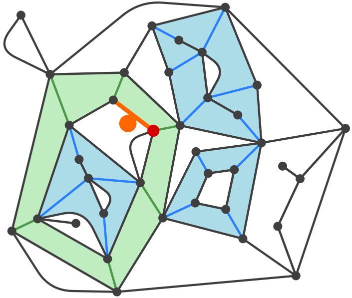

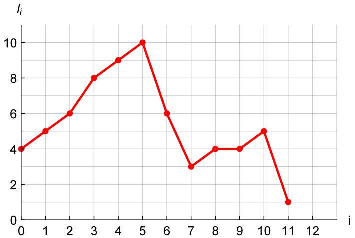

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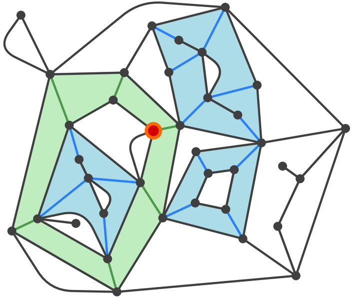

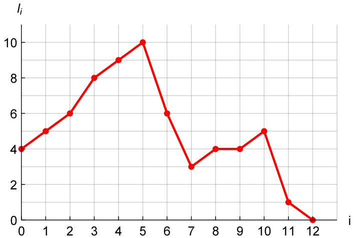

Peeling process on loop-decorated maps

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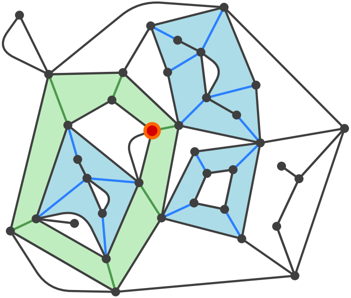

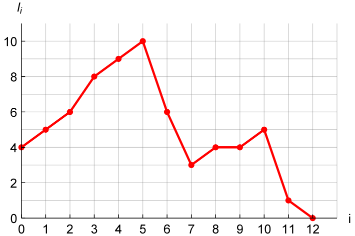

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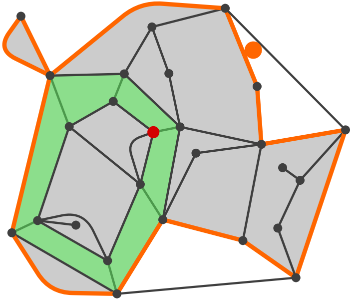
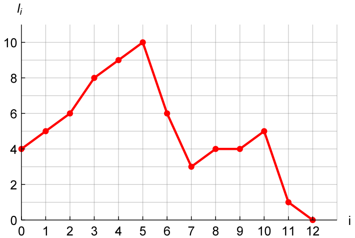
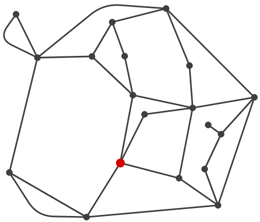
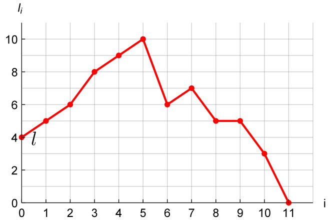
- The law of $(l_i)_i$ is not affected by taking the gasket, which is a $(\hat{\mathbf{q}}, g, n, 0, 0)$-Boltzmann loop-decorated map.





Peeling process on $\hat{\mathbf{q}}$-Boltzmann planar maps





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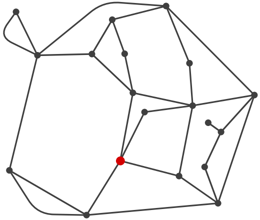

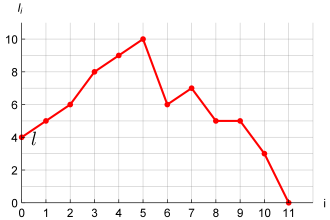

- In the absence of loops, $(l_i)_i$ is simply a biased random walk:

Proposition (TB, '15)

- *The perimeter process* $(l_i)_{i\geq 0}$ *of a* $\hat{\mathbf{q}}$*-Boltzmann planar map is given by conditioning a random walk* $(W_i)_{i\geq 0}$ *to hit 0 before hitting* $\mathbb{Z}_{<0}$.

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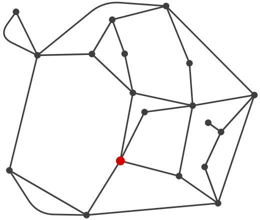

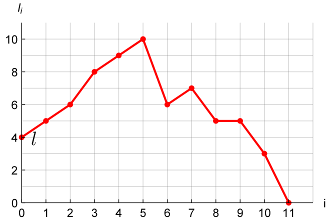

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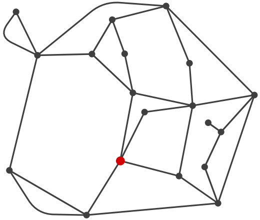
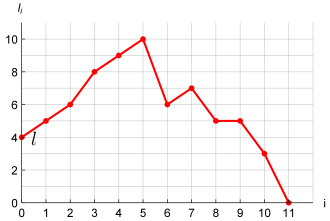
Peeling process on $\hat{\mathbf{q}}$-Boltzmann planar maps





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$$\left\{ \nu : \mathbb{P}_l\big((W_i)_i \text{ does not overshoot } 0\big) = 4^{-l}\binom{2l}{l} \right\} \longleftrightarrow \{\text{admissible } \hat{\mathbf{q}}\}.$$

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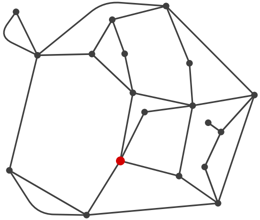

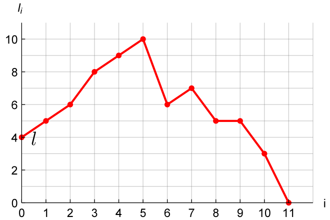

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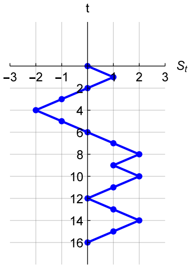
- $(l_i)_i$ *is h-transform of* $(W_i)_i$ *w.r.t.* H_0: $\mathbb{P}(l_{i+1} = l_i + k | l_i) = \frac{H_0(l_i+k)}{H_0(l_i)}\nu(k)$.

A special family of random walks

- Let $(S_t)_{t\geq 0}$ be the symmetric simple random walk started at 0



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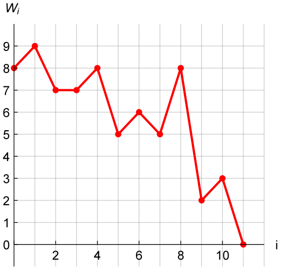

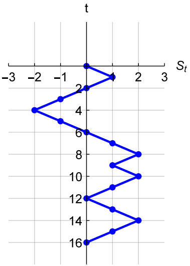

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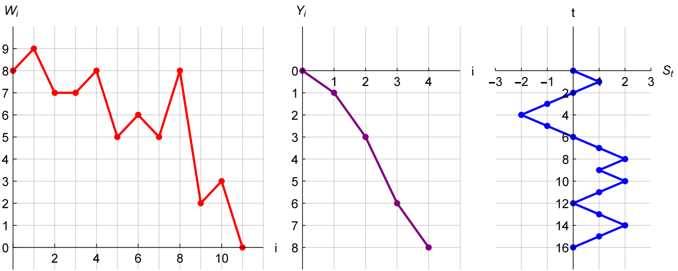

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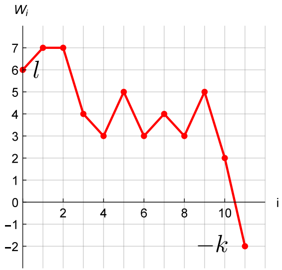

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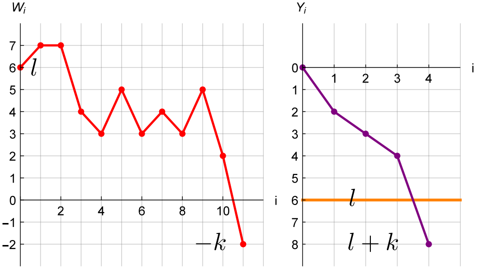

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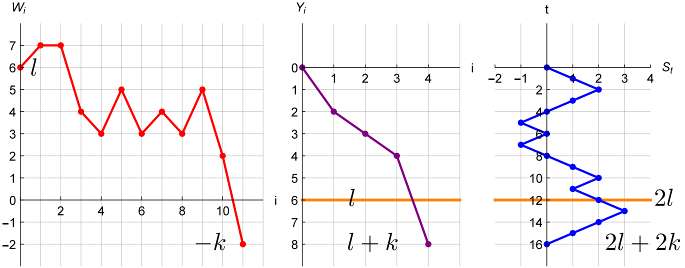

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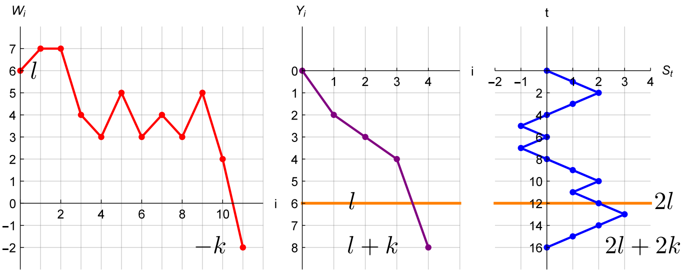

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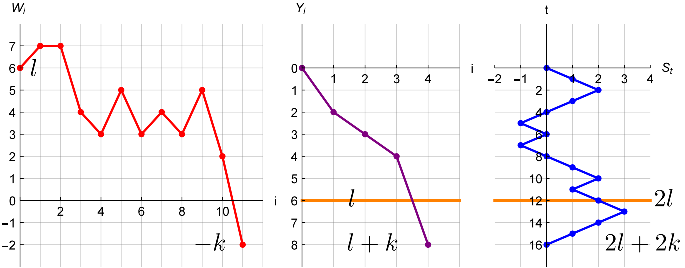

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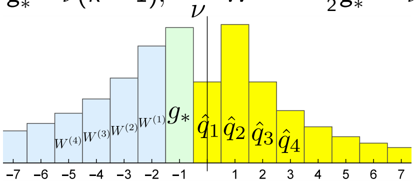
- Relation with $\hat{\mathbf{q}}$-Boltzmann planar maps:

$g_* := \frac{\nu(-1)}{2}, \quad \hat{q}_k \overset{k\geq 0}{=} g_*^{k-1}\nu(k-1), \quad W^{(l)} \overset{l\geq 0}{=} \frac{1}{2} g_*^{-l-1}\nu(-l-1),$



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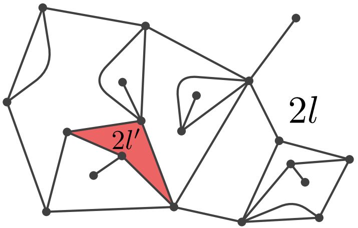
- (iv) implies that $W^{(l)} = W^{(l)}(\hat{\mathbf{q}})$ since it satisfies Tutte's equation

$$W^{(l)} = \sum_{k=1}^{\infty} \hat{q}_k W^{(l+k-1)} + \sum_{l'=0}^{l-1} W^{(l')} W^{(l-l'-1)}. \qquad (l \geq 1)$$

Building a marked Boltzmann planar map

- A *marked* $\hat{\mathbf{q}}$*-Boltzmann planar map* $\mathfrak{m} \in \mathcal{M}_{\bullet}^{(l,l')}$ is a map with root face and marked face of degree $2l > 0$ resp. $2l' \geq 0$, determined by weight $w_{\hat{\mathbf{q}}}(\mathfrak{m}) = \prod_f \hat{q}_{\deg(f)/2}$ over non-root, non-marked faces f.



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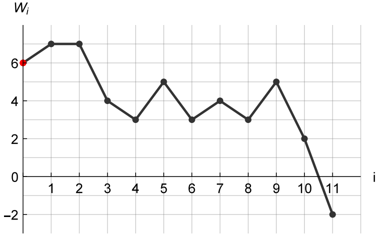

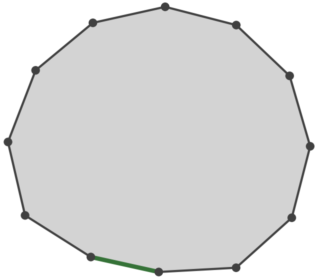

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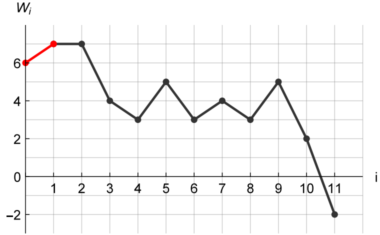

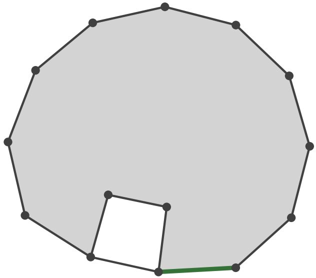

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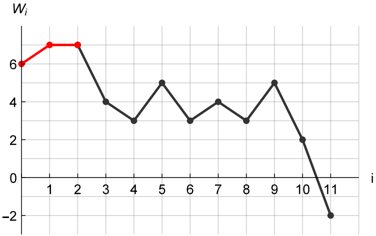

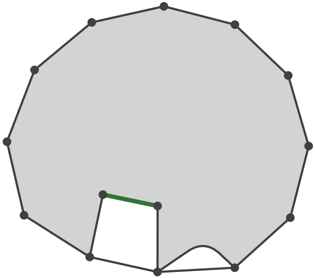

- Start with $2W_0$-gon. If $W_{i+1} \geq W_i$: insert new face, otherwise glue edges and leave a hole.

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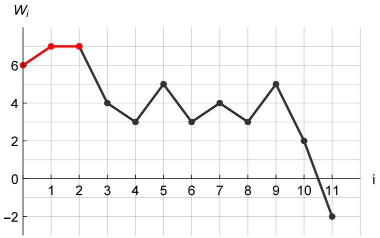

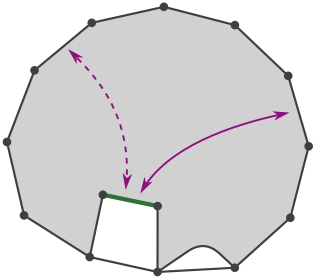

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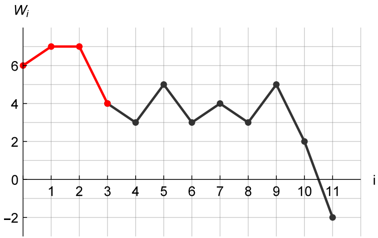

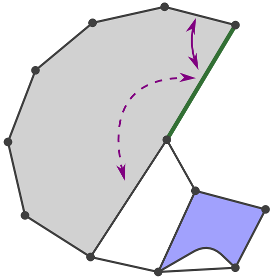

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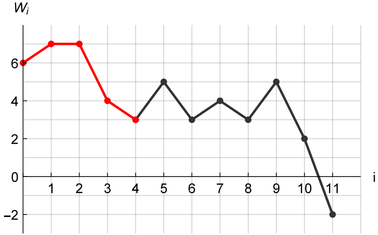

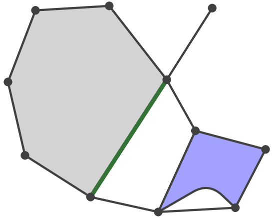

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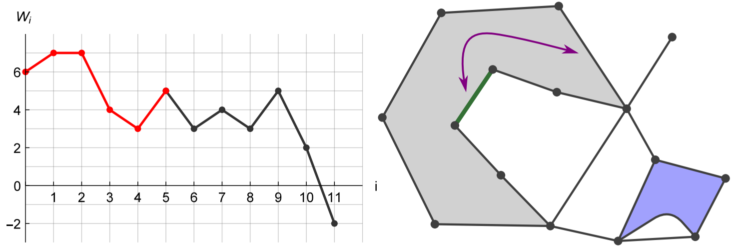

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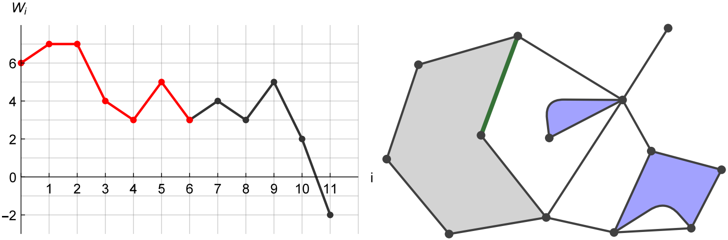

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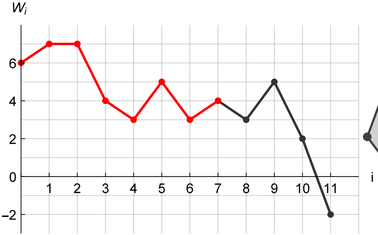

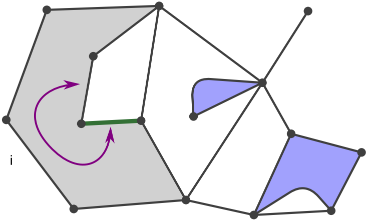

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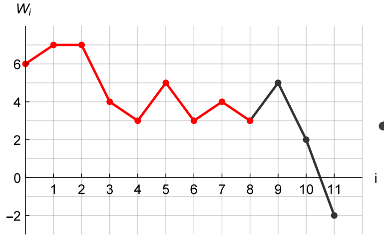

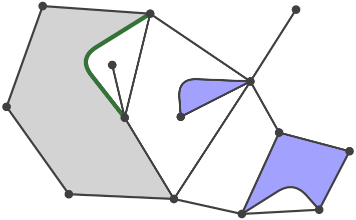

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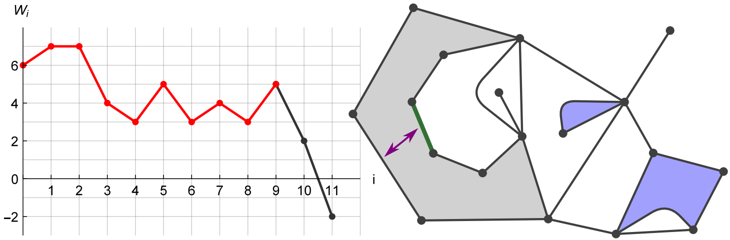

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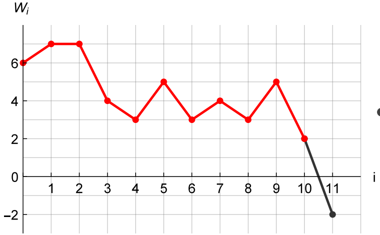

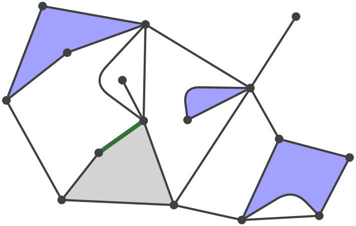

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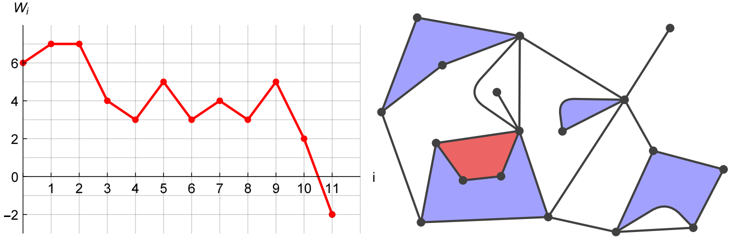

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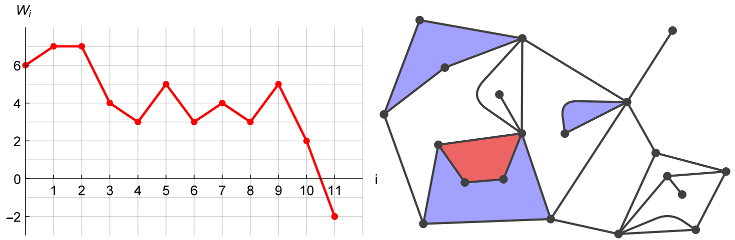

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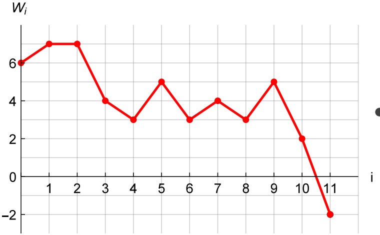

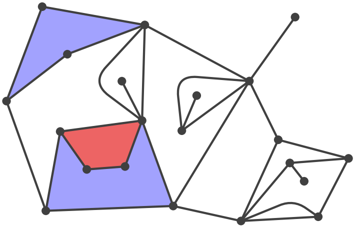

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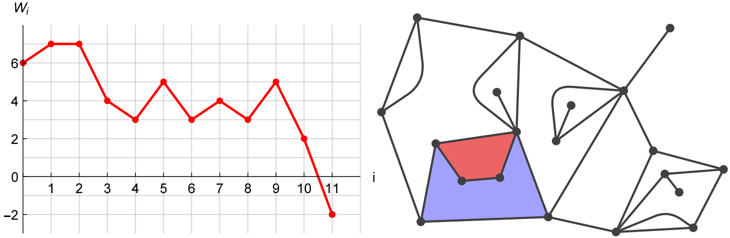

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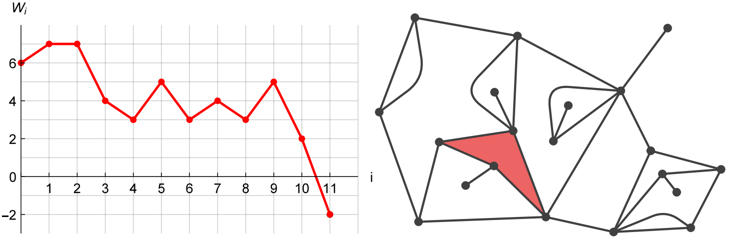

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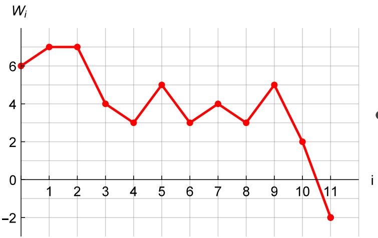

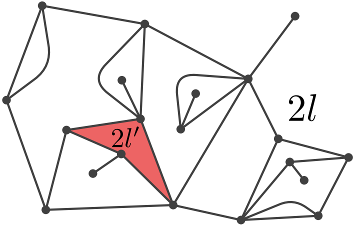

This map is a marked, rooted $\hat{\mathbf{q}}$-Boltzmann planar map $\mathfrak{m} \in \mathcal{M}_\bullet^{(l,l')}$ with independent random l' such that $\mathbb{P}(l' = k) = H_k(l)$.

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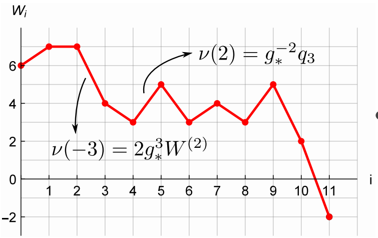

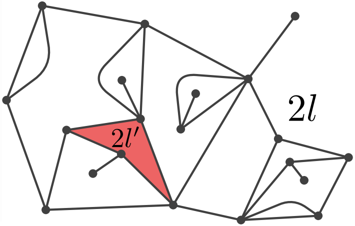

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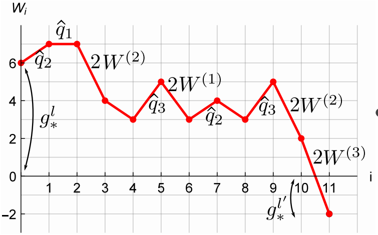

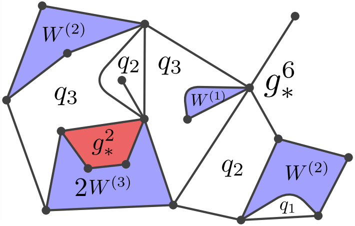

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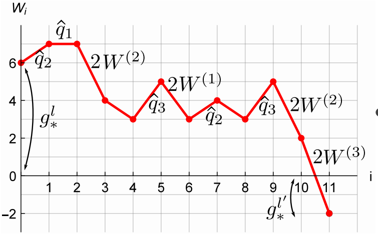

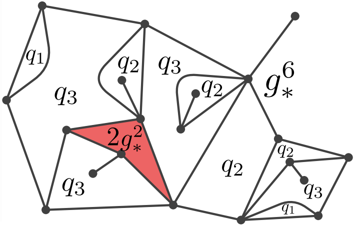

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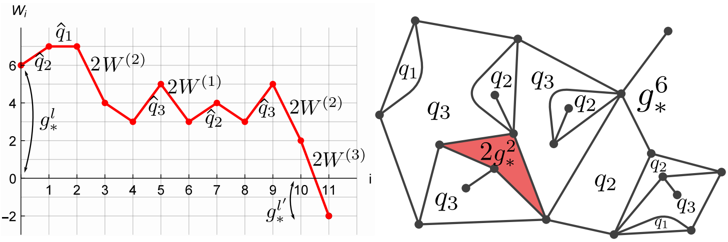
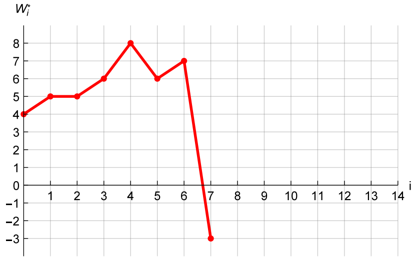
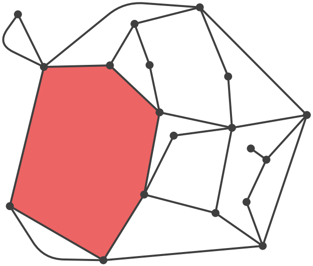


This map is a marked, rooted $\hat{\mathbf{q}}$-Boltzmann planar map $\mathfrak{m} \in \mathcal{M}_\bullet^{(l,l')}$ with independent random l' such that $\mathbb{P}(l' = k) = H_k(l)$. Conditioning on $l' = 0$ gives a marked vertex!

Partially reflected random walks

- *Reflected random walk* $(W_i^*)_i$: continue random walk $(W_i)_i$ by reflection until it hits 0.





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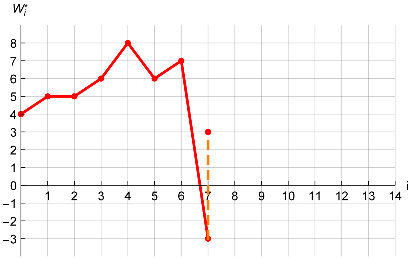

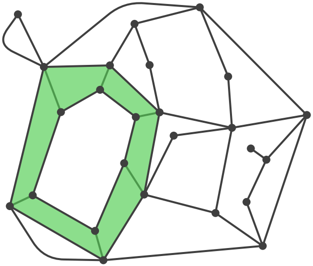

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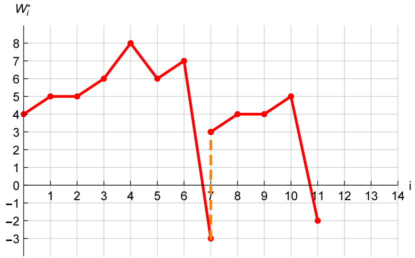

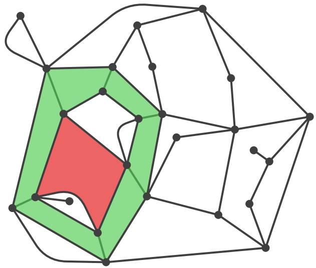

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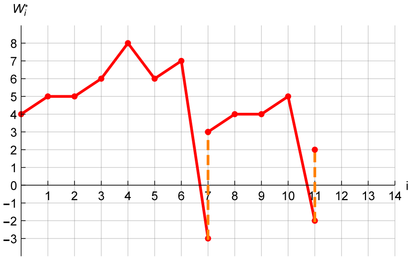

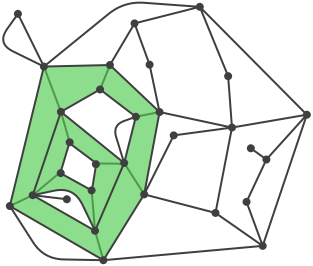

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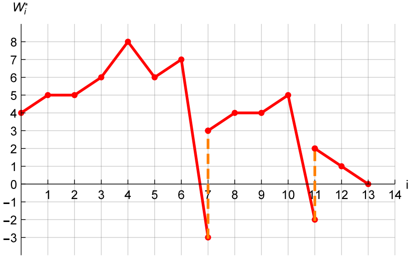

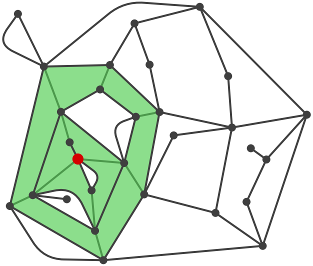

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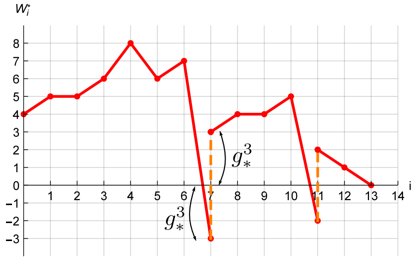

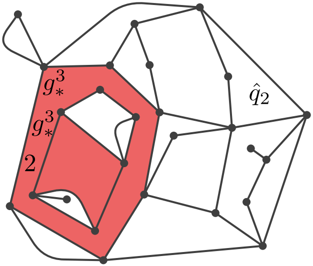

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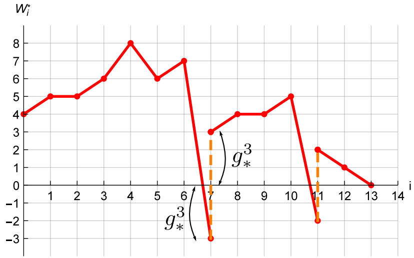
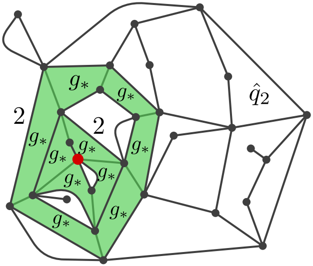
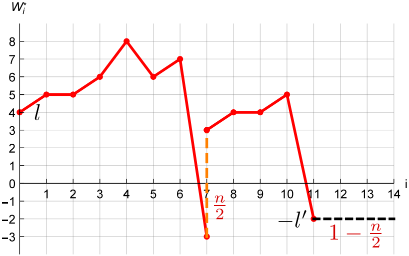


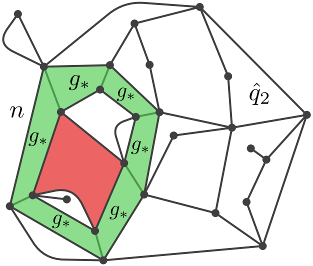


Result is a $(\hat{\mathbf{q}}, g = g_*, n = 2, 0, 0)$-Boltzmann loop-decorated map.
Critical case: increasing g or n leads to non-admissible $(\hat{\mathbf{q}}, g, n, 0, 0)$

Partially reflected random walks

- *Reflected random walk* $(W_i^*)_i$: continue random walk $(W_i)_i$ by reflection until it hits 0.
- *$\frac{n}{2}$-Partially reflected random walk* $(W_i^*)_i$: reflect with probability $\frac{n}{2}$ each time $(W_i^*)_i$ hits $\mathbb{Z}_{<0}$ and kill it otherwise.





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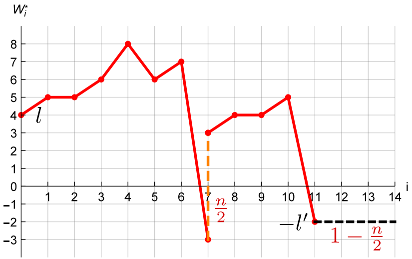
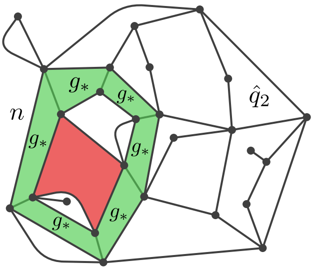
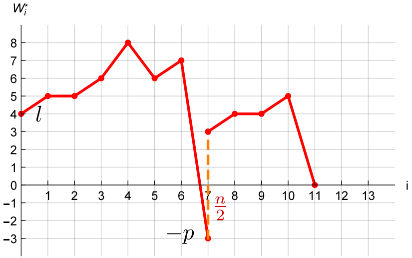




Result is a $(\hat{\mathbf{q}}, g = g_*, n, 0, 0)$-Boltzmann loop-decorated map $(\mathfrak{m}, L) \in \mathcal{LM}_{\bullet}^{(l,l')}$ with a marked face $(l' > 0)$ or vertex $(l' = 0)$, and l' is a random variable.

Partially reflected random walks (continued)

- What is the probability $h_n^{\downarrow}(l)$ that $(W_i^*)_i$ started at l is killed at 0?



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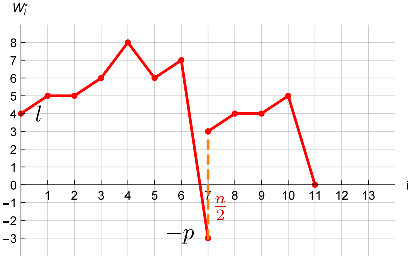

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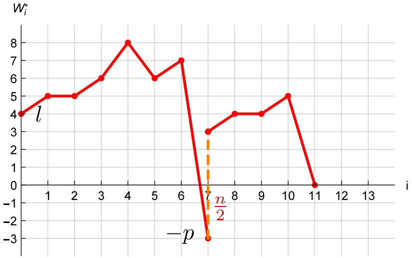
$$\sum_{l=0}^{\infty} h_n^{\downarrow}(l)\, x^{2l} = \frac{n + 2\cosh(2(b-1)\operatorname{arctanh} x)}{n+2},$$

where $b := \frac{1}{\pi}\arccos(n/2) \in [0, 1/2]$. See also [Borot, Bouttier, '15]



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The perimeter process $(l_i)_i$ of a pointed $(\hat{\mathbf{q}}, g_*, n, 0, 0)$-Boltzmann loop-decorated map is obtained by conditioning $(W_i^*)_i$ to be killed at zero, by an h-transform w.r.t. $h_n^\downarrow$, i.e.

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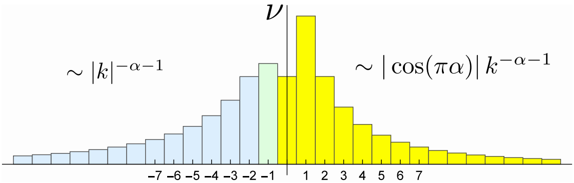
- The same is true for $(\mathbf{q}, g_*, n, \tilde{g}, \tilde{n})$-Boltzmann loop-decorated maps.

Scaling limit of the perimeter process

- First determine scaling limit of random walk $(W_i)_i$ with law ν. Recall $\nu(-k) = H_{k-1}(1) - \sum_{l=0}^{\infty} H_{k-1}(l+1)\nu(l)$.

Proposition

For our class of ν's, if ν is regularly varying, there exists $\alpha \in [1/2, 3/2]$ such that $\nu(-k) \sim k^{-\alpha-1}$ and $\frac{\nu(k)}{\nu(-k)} \to |\cos(\pi\alpha)|$.



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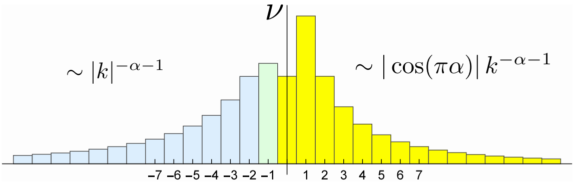

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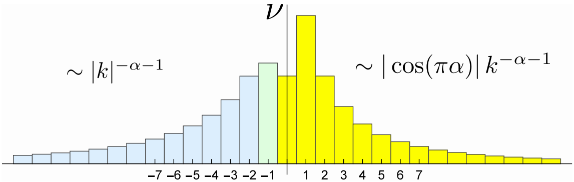
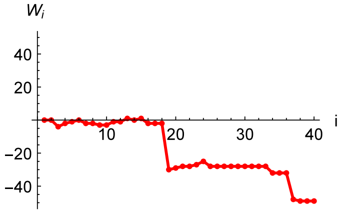
- Depending on $\mathbf{q}$: two possible values $\alpha = 1 \pm \frac{1}{\pi}\arccos(\tilde{n}/2)$ correspond to *dense* $\alpha \in (1/2, 1]$ and *dilute* $\alpha \in [1, 3/2)$ branch.



- If $\alpha \in (1/2, 3/2)$, the random walk $(W_i)_i$ has the scaling limit

$$\left(W_{\lfloor c\lambda^\alpha t\rfloor}/\lambda\right)_{t\geq 0} \xrightarrow[\lambda\to\infty]{\text{(d)}} (S_t)_{t\geq 0},$$

where $(S_t)_{t\geq 0}$ is the α-stable process with *positivity parameter* $\rho := \mathbb{P}(S_1 > 0) = 1 - 1/(2\alpha)$.



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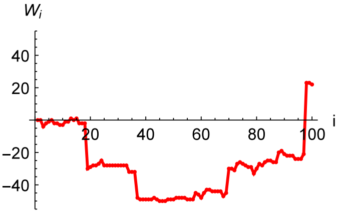

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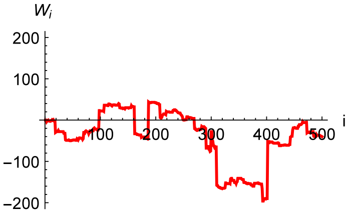

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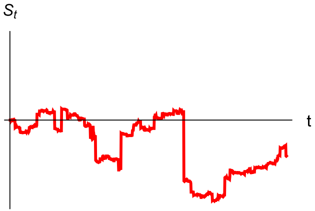

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- If $(l_i)_i$ is $(W_i)_i$ started at l_0 conditioned to not overshoot 0, then

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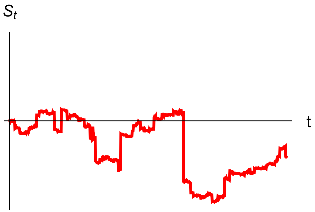
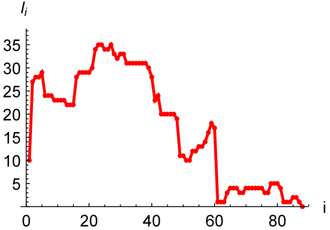
which is the α-stable process *conditioned to die continuously at 0*.

[Caravenna, Chaumont]





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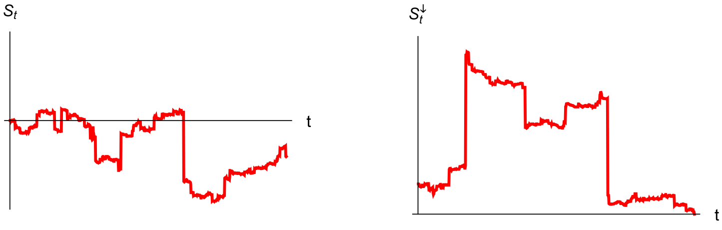

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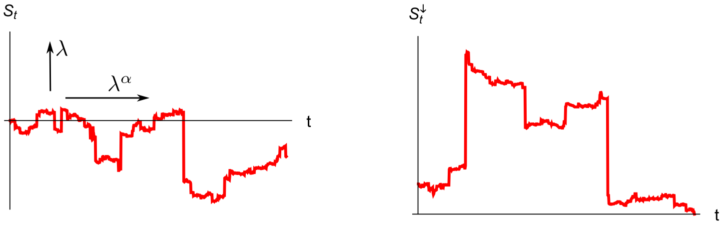
- Both are self-similar with index α.



Partially reflected stable process

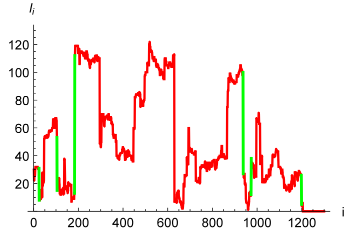
- Need to check conditions for: Markov process on $\mathbb{Z}_{>0} \xrightarrow{l_0\to\infty}$ self-similar Markov process on $(0,\infty)$. [Bertoin, Kortchemski, '14].

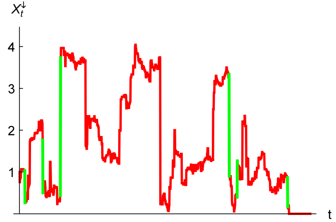
Theorem (TB, '15)

Let $n, \tilde{n} \in (0,2)$ and $\tilde{n} = -2\cos(\pi\alpha)$, $\alpha \in (1/2, 3/2)$. The perimeter $(l_i)_i$ of a $(\mathbf{q}, g_*, n, g_*, \tilde{n})$-Boltzmann loop decorated map with root face degree $2l_0$ has the scaling limit

$$\left(\frac{l_{\lfloor ct l_0^\alpha \rfloor}}{l_0}\right)_{t\geq 0} \xrightarrow[l_0\to\infty]{\text{(d)}} (X_t^{\downarrow})_{t\geq 0},$$

where $(X_t^{\downarrow})_t$ is the (self-similar) $\frac{n}{2}$-partially reflected α-stable process conditioned to die continuously at 0.





Application: integrals of $(X_t^{\downarrow})_t$.

- $(X_t^{\downarrow})_t$ is self-similar with index α and dies continuously (at $t = T_0$):

$$\int_0^{T_0} (X_t^{\downarrow})^{\gamma} \mathrm{d}t < \infty \text{ a.s.} \qquad \text{for } \gamma > -\alpha$$

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- Can determine explicitly Mellin transform in terms of Barnes double Gamma functions $G(\cdot,\cdot)$ using [Kuznetsov, Pardo, '10]

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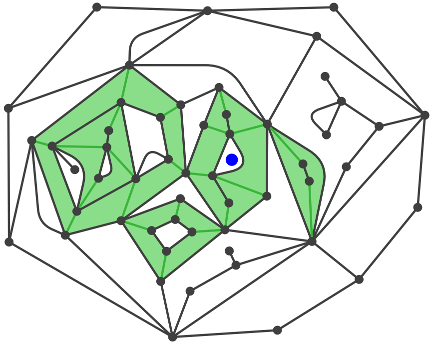
- Ugly, except when $\gamma = -1$, $n = \tilde{n} = -2\cos(\pi\alpha)$, $\alpha = 1 + \frac{1}{m}$, $m = 2, 3, \ldots$.

$$R^{\downarrow} := \int_0^{T_0} \frac{\mathrm{d}t}{X_t^{\downarrow}}, \qquad \mathbb{P}(R^{\downarrow} < r) = \frac{1}{\Gamma(\alpha)} \int_0^{\infty} \mathrm{d}Z\, Z^{\frac{1}{m}} e^{-Z} B_m\left(\frac{m}{r\, Z^{\frac{1}{m}}}\right)$$

$$B_m(y) := \frac{1 + y \cot\left(\frac{\pi}{2m}\right)}{\prod_{k=0}^{m}\left(1 - y\, i e^{i\pi k/m}\right)}$$

Distance with shortcuts (w.i.p.)

- Let d be the dual graph distance to the root with "shortcuts" in a dilute $(\mathbf{q}, n, g_*)$-Boltzmann loop-decorated map with root face $2l$.



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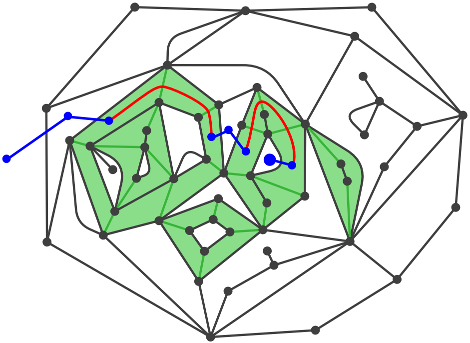

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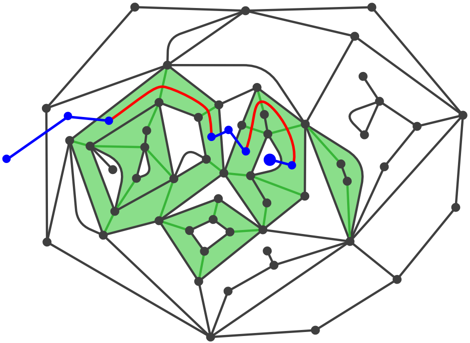

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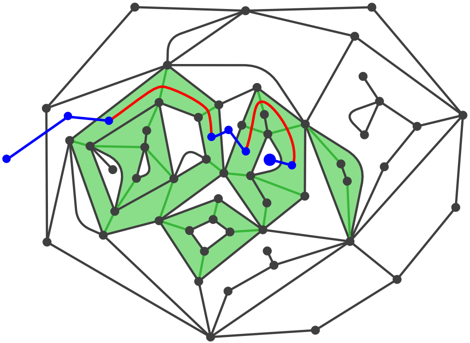

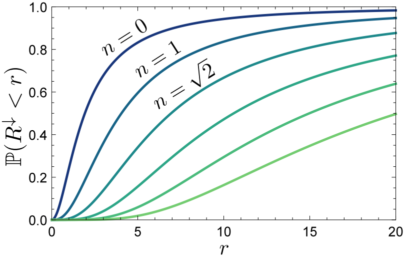

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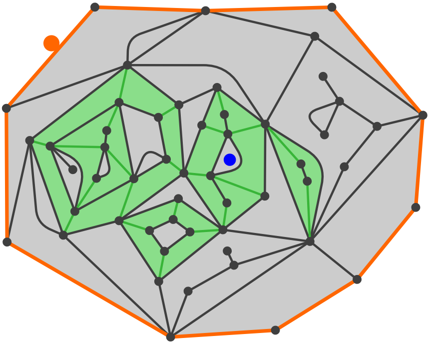

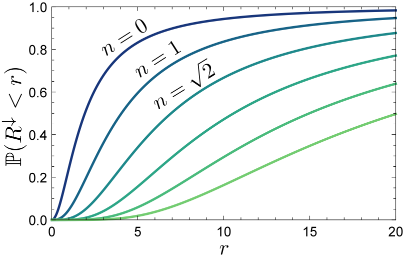

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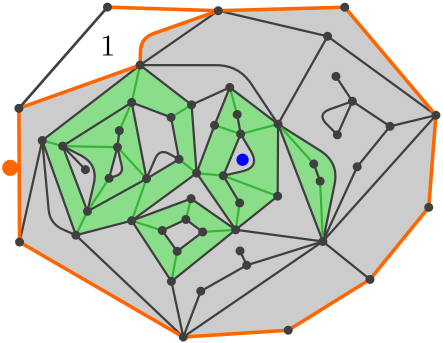

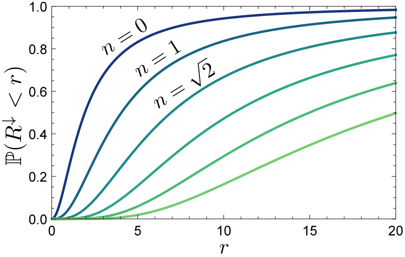

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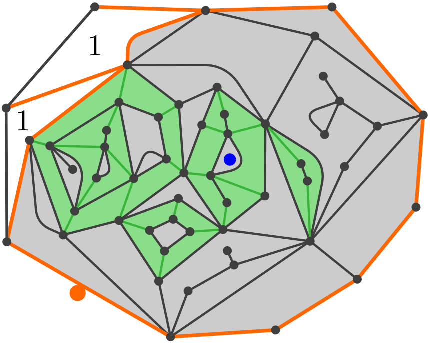

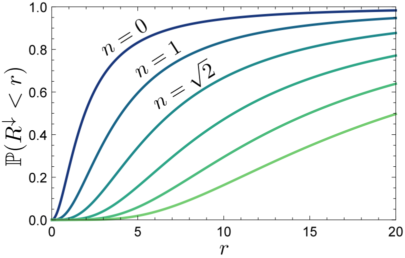

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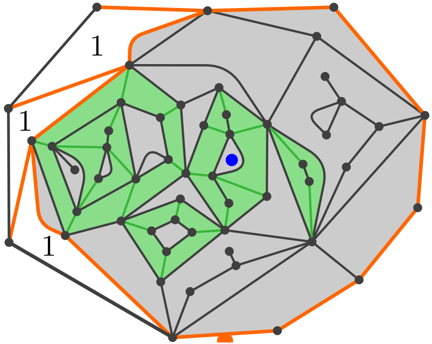

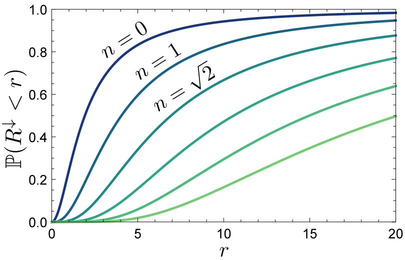

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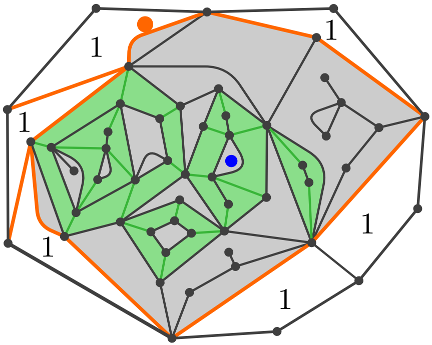

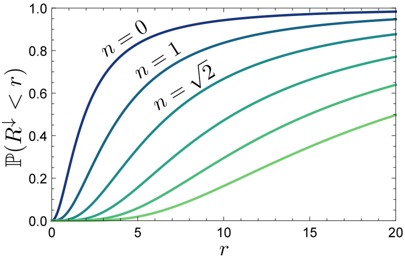

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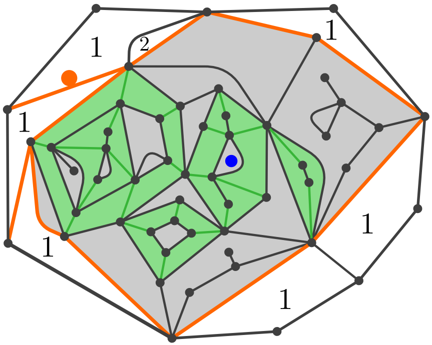

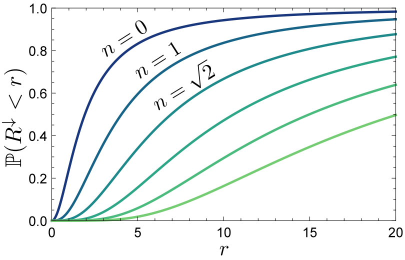

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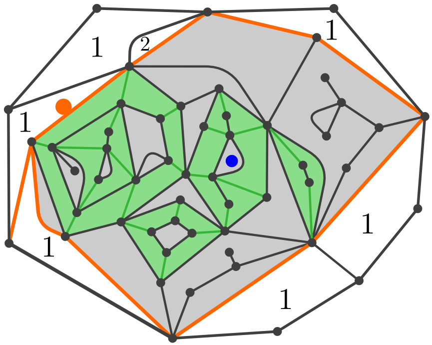

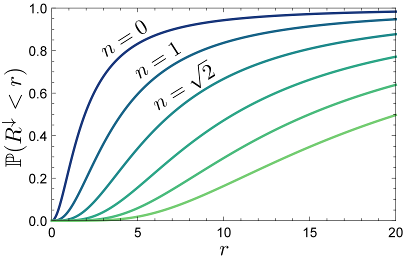

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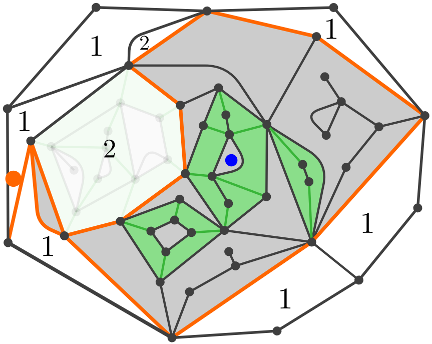

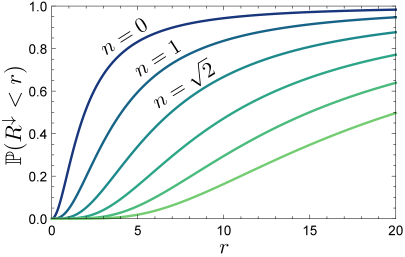

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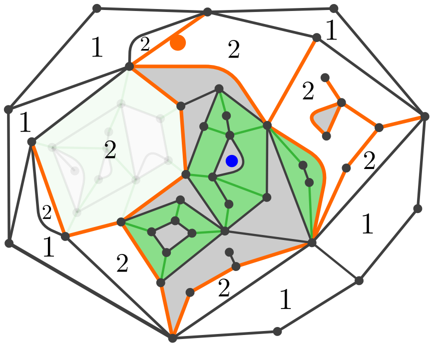

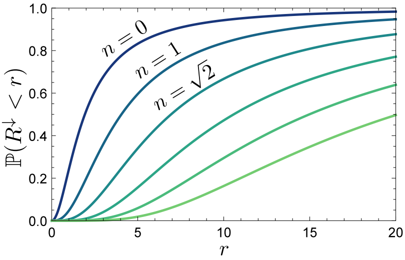

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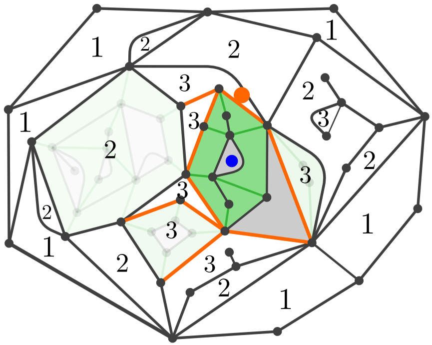

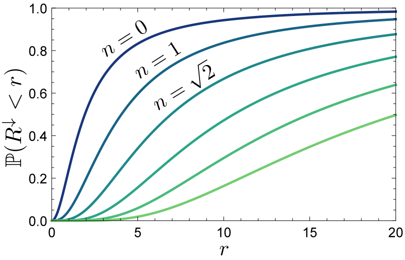

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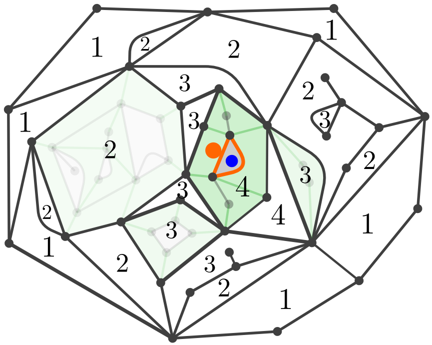

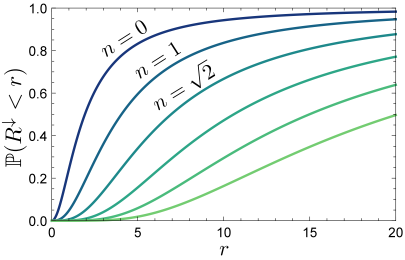

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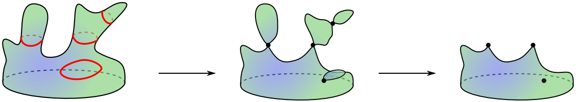

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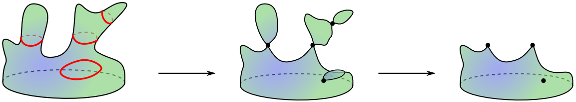

Thanks for you attention!