Baxter d-permutations and other pattern avoiding classes

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Permutations and diagrams

A **permutation** $\sigma = \sigma(1), \ldots, \sigma(n) \in S_n$ is a bijection from $[n] := \{1, 2, \ldots, n\}$ to itself.

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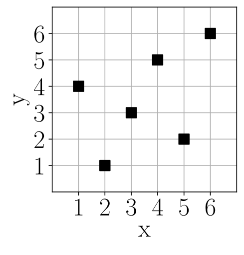
Example: the digram of $\sigma = 413526$



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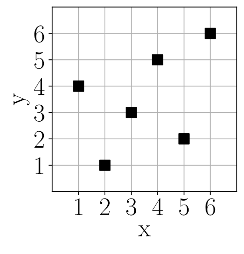


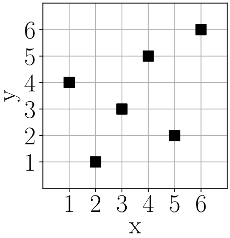
A **diagram** (of size n) is a point set on $[n] \times [n]$ with exactly 1 point per row and 1 point per column.

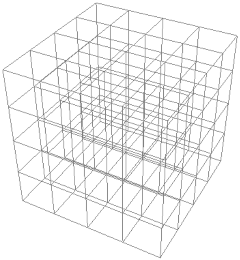
S_n the set of permutations of size n.

$|S_n| = n!$

The Starting Question

- Permutations $\Leftrightarrow$ Diagrams
- Diagrams are 2D objects
- What could be a ”3D” diagram?
- $\Rightarrow$ ”3D” Permutations?





d-Diagrams and d-Permutations

A **3-diagram** of size n is a point set on $[n]^3$ such that

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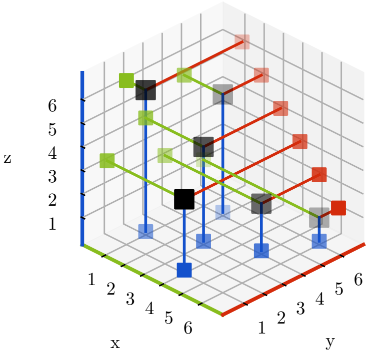
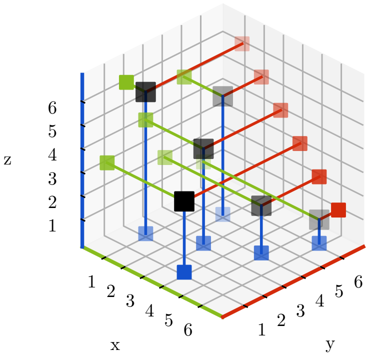


Points:
(1,2,6)
(2,5,5)
(3,3,4)
(4,1,3)
(5,4,2)
(6,6,1)

A d-**diagram** is a point set of size n on $[n]^d$ such that each **hyperplane** orthogonal $x_i = j$ with $i \in [d]$ and $j \in [n]$ contains exactly 1 point.

d-Diagrams and d-Permutations



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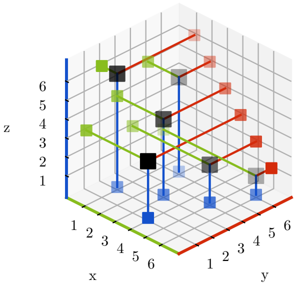
$(253146, 654321)$

A **3-permutation** $\boldsymbol{\sigma} := (\sigma_y, \sigma_z)$ is a pair of permutations.
$S_n^2 :=$ {3-permutations of size n}. $|S_n^2| = n!^2$.

A d-**permutation** of size n, $\boldsymbol{\sigma} := (\sigma_1, \ldots, \sigma_{d-1})$ is a sequence of $d-1$ permutations of size n. $S_n^{d-1} :=$ {d-permutations of size n}. $|S_n^{d-1}| = n!^{d-1}$.

Projections



$$\mathrm{proj}_{xy}(\boldsymbol{\sigma}) = \sigma_y,$$
$$\mathrm{proj}_{xz}(\boldsymbol{\sigma}) = \sigma_z,$$
$$\mathrm{proj}_{yz}(\boldsymbol{\sigma}) = \sigma_z {\sigma_y}^{-1}.$$
$$\mathrm{proj}_{yz}((253146, 654321)) = 364251.$$
$$\mathrm{proj}_{y,x}(\boldsymbol{\sigma}) = \sigma_y^{-1}.$$

Let $\overline{\boldsymbol{\sigma}} := (\mathrm{Id}_n, \sigma_1, \ldots, \sigma_{d-1})$. , the **projection** on $\boldsymbol{i}$ of d-permutation $\boldsymbol{\sigma}$ is the d'-permutation $\mathrm{proj}_{\boldsymbol{i}}(\boldsymbol{\sigma}) := \overline{\sigma}_{i_2}\overline{\sigma}_{i_1}^{-1}, \overline{\sigma}_{i_3}\overline{\sigma}_{i_1}^{-1}, \ldots, \overline{\sigma}_{i_{d'}}\overline{\sigma}_{i_1}^{-1}$. d' is the **dimension** of the projection.

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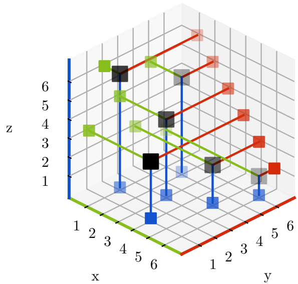

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$\boldsymbol{i} := i_1, \ldots, i_{d'} \in [d]^{d'}$, the projection $\mathrm{proj}_{\boldsymbol{i}}$ is **direct** if $i_1 < i_2 < \cdots < i_{d'}$.

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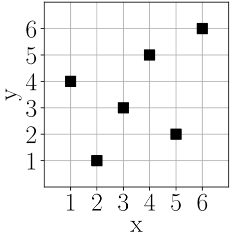
Pattern (classic)

A permutation σ **contains** a permutation (or a **pattern**) $\pi = \pi(1), \ldots, \pi(k) \in S_k$ if there exist indices $c_1 < \cdots < c_k$ such that $\sigma(c_1) \cdots \sigma(c_k)$ is order-isomorphic to π.

The set of points of indices $c_1, \cdots, c_k$ is an **occurrence** of the π.

$\sigma = 413526$ contains [several occurrences of] the pattern $\pi = 213$.



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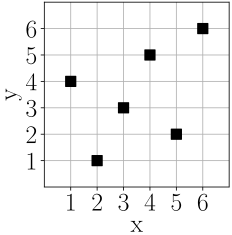

$S_n(\pi)$:= the set of permutations that avoids π.

$S_n(\pi_1, ..., \pi_k)$:= ... avoids **all** $\pi_1, \ldots, \pi_k$.

Pattern avoidance classes

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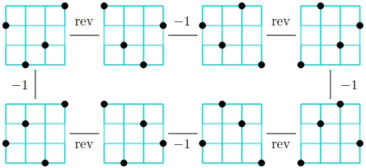
$|S_n(21)| = |S_n(12)| = 1$
π and τ are **trivially Wilf-equivalent** if there is a symmetry s of the square such that $\forall \sigma, \sigma \in S(\pi)$ iif $s(\sigma) \in S(s(\tau))$.



Pattern avoidance classes

Patterns	(1)	Sequence	Comment
12	2	$1, 1, 1, 1, 1, 1, 1, \cdots$	
12, 21	1	$1, 0, 0, 0, 0, 0, 0, \cdots$	
312	4	$\frac{1}{n+1}\binom{2n}{n} = 1, 2, 5, 14, 42, 132, \cdots$	[Knuth 73]
123	2	$\frac{1}{n+1}\binom{2n}{n} = 1, 2, 5, 14, 42, 132, \cdots$	[Knuth 73]
123, 321	1	$1, 2, 4, 4, 0, 0, 0, \cdots$	[Simion 85]
213, 321	4	$1 + \frac{n(n-1)}{2} = 1, 2, 4, 7, 11, 16, 22, \cdots$	[Simion 85]
312, 231	2	$2^{n-1} = 1, 2, 4, 8, 16, 32, 64, \cdots$	[Simion 85]
231, 132	4	$2^{n-1} = 1, 2, 4, 8, 16, 32, 64, \cdots$	[Simion 85]
312, 321	4	$2^{n-1} = 1, 2, 4, 8, 16, 32, 64, \cdots$	[Simion 85]
213, 132, 123	2	Fibonacci: $1, 2, 3, 5, 8, 13, 21, \cdots$	[Simion 85]
231, 213, 321	8	$n = 1, 2, 3, 4, 5, 6, 7, \cdots$	[Simion 85]
312, 132, 213	4	$n = 1, 2, 3, 4, 5, 6, 7, \cdots$	[Simion 85]
312, 321, 123	4	$1, 2, 3, 1, 0, 0, 0, \cdots$	
321, 213, 123	4	$1, 2, 3, 1, 0, 0, 0, \cdots$	
321, 213, 132	2	$n = 1, 2, 3, 4, 5, 6, 7, \cdots$	[Simion 85]

(1): Number of trivially Wilf-Equivalent patterns.

patterns and d-permutations

$\boldsymbol{\sigma}$ **contains** a pattern $\boldsymbol{\pi}$ if

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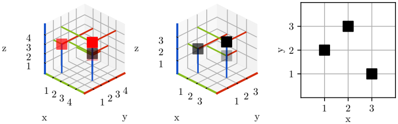

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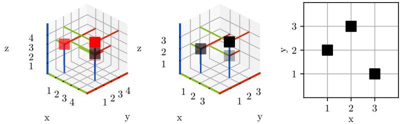
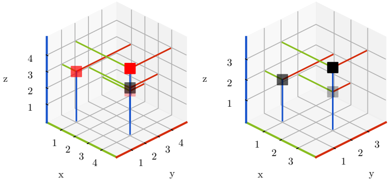


$(1432, 3124)$ contains the pattern $(132, 213)$.

$\boldsymbol{\sigma}$ **contains** a pattern π_1 if $\text{proj}_{xy}, \text{proj}_{xz}$ or proj_{yz} contains π_1.

patterns and d-permutations

Let $\boldsymbol{\sigma} \in S_n^{d-1}$ -and $\boldsymbol{\pi} \in S_k^{d'-1}$ with $k \leq n$. Then $\boldsymbol{\sigma}$ **contains the pattern** $\boldsymbol{\pi}$, if there exist a **direct** projection $\boldsymbol{\sigma}' = \mathrm{proj}_{\boldsymbol{i}}(\boldsymbol{\sigma})$ of dimension d' that contains $\boldsymbol{\pi}$.



$(1432, 3124)$ contains the pattern $(132, 213)$ and the pattern 231.

$\boldsymbol{\sigma}$ **contains the pattern** $\boldsymbol{\pi}$ if there are indices $c_1 < \cdots < c_k$ such that $\sigma'_i(c_1) \cdots \sigma'_i(c_k)$ is order-isomorphic to π_i for all $i \in [d']$.

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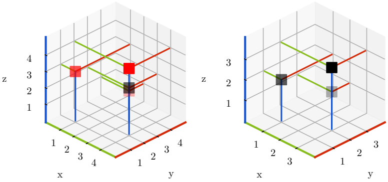

$(1432, 3124)$ contains the pattern $(132, 213)$ and the pattern 231.

Remark 1: $(132, 312)$ doesn't contain $(12, 12)$ but 132 and 312 both contain the pattern 12 (but on different positions).

$\boldsymbol{\sigma}$ **contains the pattern** $\boldsymbol{\pi}$ if there are indices $c_1 < \cdots < c_k$ such that $\sigma_i'(c_1) \cdots \sigma_i'(c_k)$ is order-isomorphic to π_i for all $i \in [d']$.

3-Pattern avoidance classes

Patterns	(1)	Sequence	Comment
$(12, 12)$	4	$1, 3, 17, 151, 1899, 31711, \cdots$	weak-Bruhat interv
$(12, 12), (12, 21)$	6	$n! = 1, 2, 6, 24, 120 \cdots$	$\sigma_1 \Rightarrow \sigma_2$
$(12, 12), (12, 21), (21, 12)$	4	$1, 1, 1, 1, 1, 1, \cdots$	1 diagonal
$(12, 12), (12, 21), (21, 12), (21, 21)$	1	$1, 0, 0, 0, 0, 0, \cdots$	
$(123, 123)$	4	$1, 4, 35, 524, 11774, 366352, 14953983, \cdots$	*new*
$(123, 132)$	24	$1, 4, 35, 524, 11768, 365558, 14871439, \cdots$	*new*
$(132, 213)$	8	$1, 4, 35, 524, 11759, 364372, 14748525, \cdots$	*new*
$(12, 12), (132, 312)$	48	$(n+1)^{n-1} = 1, 3, 16, 125, 1296 \cdots$	**[Atkinson et al. 9**
$(12, 12), (123, 321)$	12	$1, 3, 16, 124, 1262, 15898, \cdots$	distributive lattice
$(12, 12), (231, 312)$	8	$1, 3, 16, 122, 1188, 13844, \cdots$	*A295928?*

(1): Number of trivially Wilf-Equivalent patterns.

2-Pattern avoidance classes

Patterns	(1)	Sequence	Comment
12	1	$1, 0, 0, 0, 0, \cdots$	unvoidable pattern
21	1	$1, 1, 1, 1, 1, \cdots$	1 diagonal
123	1	$1, 4, 20, 100, 410, 1224, 2232, \cdots$	*new*
132	2	$1, 4, 21, 116, 646, 3596, 19981, \cdots$	*new*
231	2	$1, 4, 21, 123, 767, 4994, 33584, \cdots$	*new*
321	1	$1, 4, 21, 128, 850, 5956, 43235, \cdots$	*new*
$123, 132$	2	$1, 4, 8, 8, 0, 0, 0, \cdots$	
$123, 231$	2	$1, 4, 9, 6, 0, 0, 0, \cdots$	
$123, 321$	1	$1, 4, 8, 0, 0, 0, 0, \cdots$	
$132, 213$	1	$1, 4, 12, 28, 58, 114, 220, \cdots$	[**Sun22+**]
$132, 231$	4	$1, 4, 12, 32, 80, 192, 448, \cdots$	$A001787$[**Sun22+**]
$132, 321$	2	$1, 4, 12, 27, 51, 86, 134, \cdots$	$A047732$[**Sun22+**]
$231, 312$	1	$1, 4, 10, 28, 76, 208, 568, \cdots$	$A026150$[**Sun22+**]
$231, 321$	2	$1, 4, 12, 36, 108, 324, 972, \cdots$	$A003946$[**Sun22+**]

(1): Number of trivially Wilf-Equivalent patterns.

1– and 2-Patterns avoidance classes

Patterns	(1)	Sequence	Comment
12, (12, 12)	1	$1, 0, 0, 0, 0, \cdots$	12
12, (21, 12)	3	$1, 0, 0, 0, 0, \cdots$	12
21, (12, 12)	1	$1, 0, 0, 0, 0, \cdots$	
21, (21, 12)	3	$1, 1, 1, 1, 1, \cdots$	21
123, (12, 12)	1	$1, 3, 14, 70, 288, 822, 1260, \cdots$	*new*
123, (12, 21)	3	$1, 3, 6, 6, 0, 0, 0, \cdots$	
132, (12, 12)	2	$1, 3, 11, 41, 153, 573, 2157, \cdots$	*A0281593?*
132, (12, 21)	6	$1, 3, 11, 43, 173, 707, 2917, \cdots$	*A026671?*
231, (12, 12)	2	$1, 3, 9, 26, 72, 192, 496, \cdots$	*A072863?*
231, (12, 21)	4	$1, 3, 11, 44, 186, 818, 3706, \cdots$	*new*
231, (21, 12)	2	$1, 3, 12, 55, 273, 1428, 7752, \cdots$	*A001764?*
321, (12, 12)	1	$1, 3, 2, 0, 0, 0, 0, \cdots$	
321, (12, 21)	3	$1, 3, 11, 47, 221, 1113, 5903, \cdots$	*A217216?*

(1): Number of trivially Wilf-Equivalent patterns.

Separable permutations Sep_n

direct sum $\sigma \oplus \pi$: add π in the top right corner of σ.
skew sum $\sigma \ominus \pi$: add π in the bottom right corner of σ.
separable: size 1 or a direct/skew sum separable permutations.

σ and π two permutations respectively of size n and k.

$$\sigma \oplus \pi := \sigma(1), \dots, \sigma(n), \pi(1) + k, \dots, \pi(k) + n \text{ and}$$

$$\sigma \ominus \pi := \sigma(1) + k, \dots, \sigma(n) + k, \pi(1), \dots, \pi(k).$$

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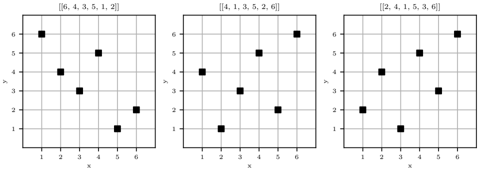

On the left the separable permutation
$643512 = 1 \ominus ((1 \ominus 1) \oplus 1) \ominus (1 \oplus 1)$.

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[Brightwell 92]:

$$|Sep_n| = \frac{1}{n-1} \sum_{k=0}^{n-2} \binom{n-1}{k} \binom{n-1}{k+1} 2^{n-k-1}.$$

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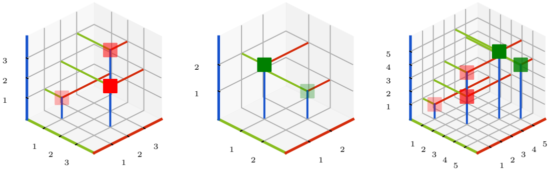
$Sep_n = S_n(2413, 3142)$ **[Bose Buss Lubiw 98]**

Separable d-permutations [Atkinson Mansour 10]

separable d-permutation: size 1 or a d-sum separable permutations.



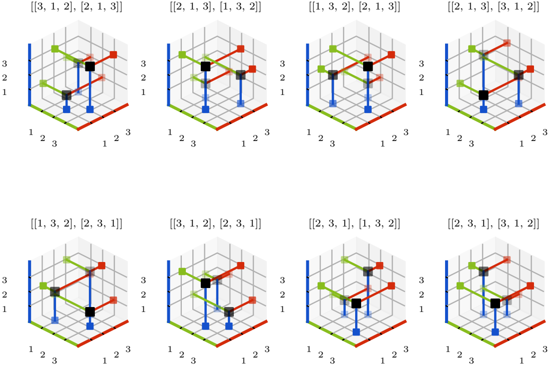
$$p_1 = ({\color{blue}132}, {\color{red}132}) = ({\color{blue}1}, {\color{red}1}) \oplus^{(+++)} (({\color{blue}1}, {\color{red}1}) \oplus^{(+--)} ({\color{blue}1}, {\color{red}1}))$$

Let $\boldsymbol{\sigma}$ and $\boldsymbol{\pi}$ two d-permutations and $\mathrm{dir} \in \{+,-\}^d$. The **$d$-sum** with respect to direction dir is :

$$\boldsymbol{\sigma} \oplus^{\mathrm{dir}} \boldsymbol{\pi} := \overline{\sigma}_2 \oplus_2^{\mathrm{dir}} \overline{\pi}_2, \dots, \overline{\sigma}_d \oplus_d^{\mathrm{dir}} \overline{\pi}_d,$$

where $\oplus_i^{\mathrm{dir}}$ is $\oplus$ if $\mathrm{dir}_i = +$ and $\ominus$ if $\mathrm{dir}_i = -$.

Separable d-permutations [Atkinson Mansour 10]



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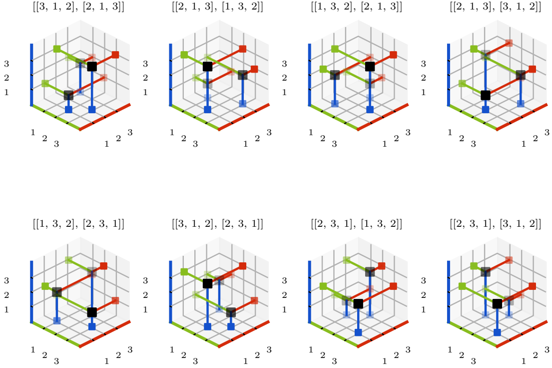

[Atkinson Mansour 10]

$$Sep_n^{d-1} = S_n^{d-1}(\mathrm{Sym}((132, 213)), 2413, 3142)$$

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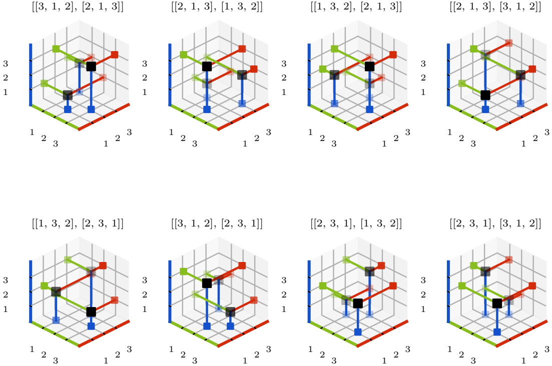

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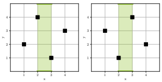
[Atkinson Mansour 10]

$$|Sep_n^{d-1}| = \frac{1}{n-1}\sum_{k=0}^{n-2}\binom{n-1}{k}\binom{n-1}{k+1}(2^{d-1}-1)^k(2^{d-1})^{n-k-1}.$$

Vincular Patterns and Baxter permutations

vincular pattern :a pattern where some entries must be consecutive in the permutation (**adjacencies**). Ex: $2413|_2$ and $3142|_2$



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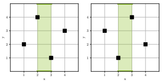

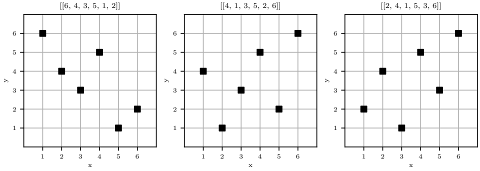

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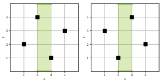
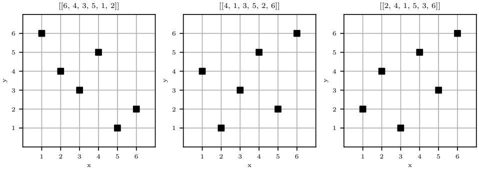
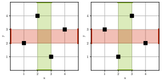




Baxter permutations: $B_n := S_n(2413|_2, 3142|_2)$.

Generalized vincular Patterns

generalized vincular pattern : horizontal and/or vertical adjacencies. Ex: $2413|_{2,2}$ and $3142|_{2,2}$.



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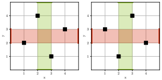

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Natural extension of generalized vincular patterns to d-permuations...

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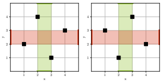

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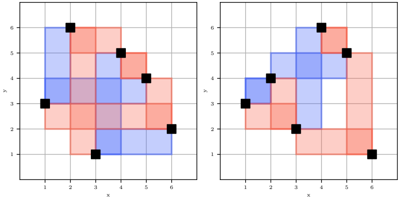
Natural extension of generalized vincular patterns to d-permuations...

But what could be a Baxter d-permutation...

Well sliced permutations

A **slice** is a rectangle defined by two adjacent points. **type**: horizontal or vertical. The **direction** of a slice is: + or **-**.



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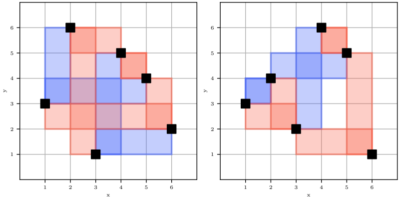

well-sliced: each slice intersects exactly 1 slice of each type and two intersecting slices share the same direction.

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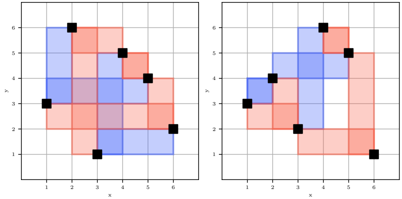

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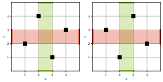
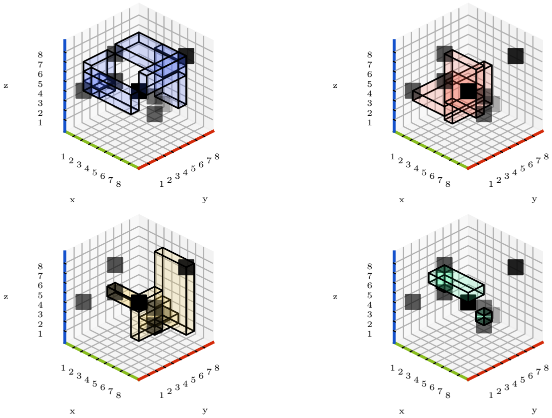
Proposition: Baxter $\equiv$ well-sliced



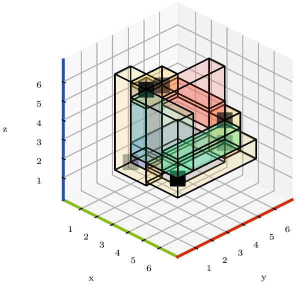
Well-sliced d-permutations

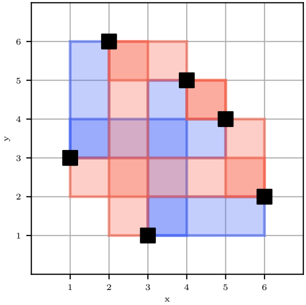
The **direction** of a slice is: ++, +-, -+, –. The **type** of a slice is: x, y or z.



A **Baxter d-permutation** is a d-permutation such that each of its $d' \leq d$ projection is well-sliced.

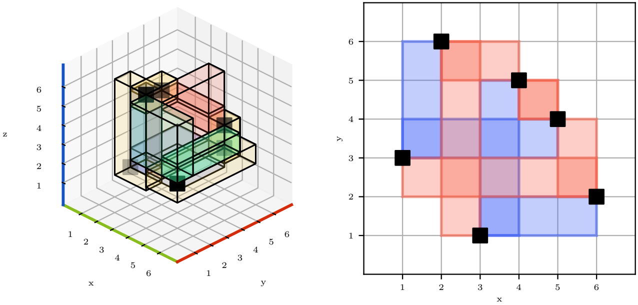
Well-sliced d-permutations vs Baxter d-permutation





well-sliced but not Baxter.

Baxter d-permutation



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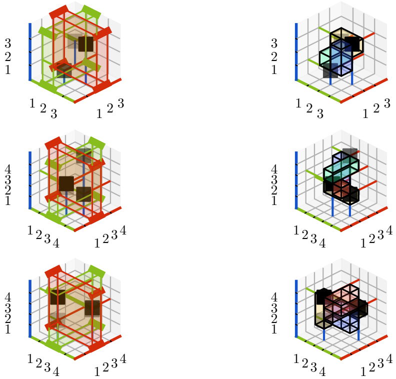
Baxter d-permutation characterisation



Theorem

$B_n^{d-1} = S_n^{d-1}(\mathrm{Sym}(2413|_{2,2}), \mathrm{Sym}((312, 213)|_{1,2,.}),$
$\mathrm{Sym}((3412, 1432)|_{2,2,.}), \mathrm{Sym}((2143, 1423)|_{2,2,.})).$

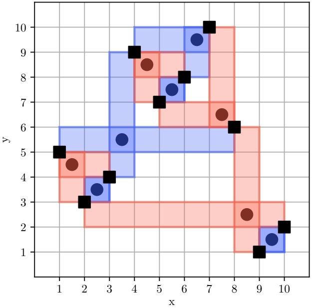
Baxter d-permutation enumration

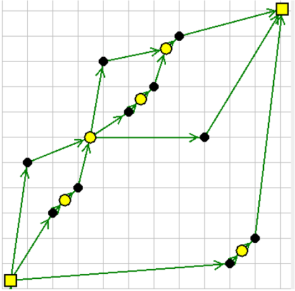
$$|B_n^{d-1}|$$

n/d	2	3	4	5
1	1	1	1	1
2	2	4	8	16
3	6	28	120	496
4	22	260	2440	20816
5	92	2872	59312	1035616
6	422	35620		
7	2074	479508		

open problem: Enumeration formula?

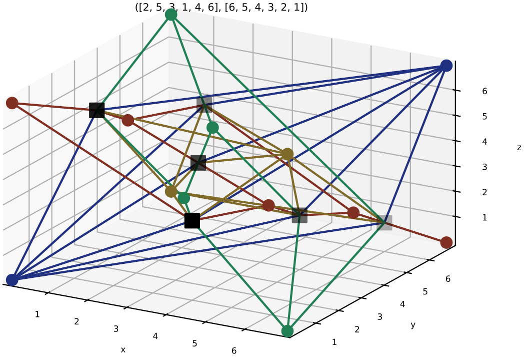
Maps ?





[Bonichon Bousquet Fusy 10] There is a bijection between Baxter permutations and plane bipolar orientations.

Maps ?



conclusion/perspectives

- nice framework
- nice generalisation of Baxter permutation
- lot of open problems.
- implementation available
  `plmlab.math.cnrs.fr/bonichon/multipermutation`

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Have fun !